

Multidisciplinary Design Optimization of Air-based Battery Thermal Management System in Electric Vehicles

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Thermal management plays a significant role in the lifetime and safety of lithium-ion battery modules in electric vehicles (EVs). To enhance the performance of a battery thermal management system (BTMS), a multidisciplinary design optimization (MDO) model of BTMS is developed in this paper. A computational fluid dynamics (CFD) based method is first developed to analyze the effects of key design variables (i.e., heat flux, mass flow rate, and passage spacing size) on the performance of BTMS. To perform optimization based on the high-fidelity CFD model, surrogate models are developed to represent the BTMS performance metrics (i.e., pressure difference between air inlet and outlet and maximum temperature difference among battery cells) as functions of key design parameters. The MDO results show that the BTMS power consumption is reduced by 49.34% and the temperature difference is reduced by 17.91%.

I. Introduction

ELECTRIC vehicles (EVs) have been developing rapidly in the past two decades. Due to the narrow range of the optimal operating temperature, the battery safety becomes one of challenges in EVs.¹ Aiming to improve the thermal performance of batteries, many cooling schemes, such as air-based systems, liquid-based systems, phase change material-based systems, heat pipe-based systems, or combination systems, have been developed as battery thermal management systems (BTMS).²⁻⁶ The air-based BTMS has been widely applied in EVs due to its low cost, simple structure, and light weight.

Most of the existing work in air-based BTMS focuses on cooling channel design and optimization. For example, Park⁷ simulated the cooling performance with different types of air ventilations, and found that the cooling performance was significantly improved by adding an extra ventilation. Hwang et al.⁸ has built a computational fluid dynamics (CFD) model of an air-based BTMS, and found that the locations and shapes of the air inlet and outlet had significant impacts on the heat dissipation. Fan et al.⁹ studied the effects of gap spacing and air flow rate on the cooling performance, and found that one-side cooling

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and uneven gap spacing could improve the temperature uniformity. Zhao et al.¹⁰ established air cooling models for a cylindrical lithium-ion power battery pack to study the thermal management performance with different parameters, such as ventilation types and velocities, gap spacing between battery cells, temperatures of environment and inlet air, number of single row cells, and battery dimensions. Xun et al.¹¹ developed numerical models of BTMS to analyze the impacts of the volume ratio and the size of the cooling channel, and found that a larger channel could improve the evenness of the temperature distribution and the energy efficiency. Ji et al.¹² proposed an active temperature control method by optimally arranging battery cells and controlling the coolant flow rate to equalize the temperature distribution of the battery cells.

The combination of electrochemical and thermal models has also been investigated in the literature. Pesaran et al.¹³ developed an electro-thermal finite element approach to predict the thermal performance of a battery module based on realistic geometry, material properties, loads, and boundary conditions. Amiribavandpour et al.¹⁴ constructed an improved theoretical electrochemical-thermal model of battery packs in EVs to study the impacts of current discharges and ambient temperatures on the battery cell temperature. Li et al.¹⁵ used a multi-scale and multi-dimensional approach to solve the Newman's pseudo-2D porous electrode-scale model in a battery, and a linear approximation was used to improve the efficiency of CFD simulations.

Additionally, the whole battery thermal management system, including air supply sub-system, battery cells, battery module, and other components, is also studied in the literature. Hamut et al.¹⁶ conducted an energetic environmental analysis of hybrid electric vehicle thermal management systems to determine energy destruction rates and energy efficiencies.

Overall, most of the existing work on BTMS focuses on only one part of the battery system. However, the performance of battery is affected by a number of different components. Generally, a typical air-based BTMS consists of an air supply system and battery cells. The battery cells supply the power to the vehicle motor as well as the air supply system, accompanied with heat generation. The air supply system provides the air to take away the heat and to cool the battery cells within the battery module. There exist a number of challenges to design an optimal air-based BTMS. First, the heat dissipation performance is critical to the lifetime of a battery. The optimal operational temperature of the battery is generally within a small range. If the battery cell temperature exceeds this allowable range, the lifetime of the battery will be significantly shortened. Under extreme situations, the EV may be destroyed by a battery cell burning or explosion due to an excessive high temperature. Second, the power consumption of the cooling system is expected to be low, since the power is also provided the battery. Third, a small size of the battery module is expected, which determines how many battery modules to be used in an EV. Lastly, the heat generation is varying with the vehicle speed. Thus, it is important to maintain a stable thermal environment for the battery cells at varying speeds.

To address all the challenges discussed above, a multidisciplinary design optimization (MDO) framework is developed in this paper to study the coupling effects among different sub-systems (i.e., air supply system, battery cell, and battery module) in the BTMS of lithium-ion batteries. The remaining of the paper is organized as follows. Section II describes a general air-based BTMS; the CFD modeling and MDO of BTMS are developed in Section III; Section IV discusses the BTMS optimization results; Concluding remarks and future work are provided in Section V.

II. Air-based BTMS

Figure 1 shows an air-based BTMS scheme with a lithium-ion battery module, which contains an air supply system and 36 lithium-ion prismatic cells. The dimension of a single cell is 65mm×151mm×16mm. The battery cells are positioned vertically inside the module with evenly spaced cooling passages between neighboring cells. A cooling fan positioned on the side of the module is used to provide the cooling air to improve the thermal environment of the battery module. The cells are numbered 1 to 36 in sequence while the passages are numbered 1 to 37, with 1 denoting the cell or the passage closest to the fan. The air steam flows into the inlet manifold that is at the bottom of the battery module, and passes through the 37 passages. After cooling the battery cells, the air converges in the outlet manifold and flows out to the atmosphere. The height of the manifold is 20 mm. Note that, the inlet and outlet are on the same side of the battery module.

In the BTMS system, the fan is powered by the battery module. The ideal fan power (P_{Fan}) can be calculated by the volume flow rate (\dot{V}) and the pressure difference between the air inlet and outlet (Δp),

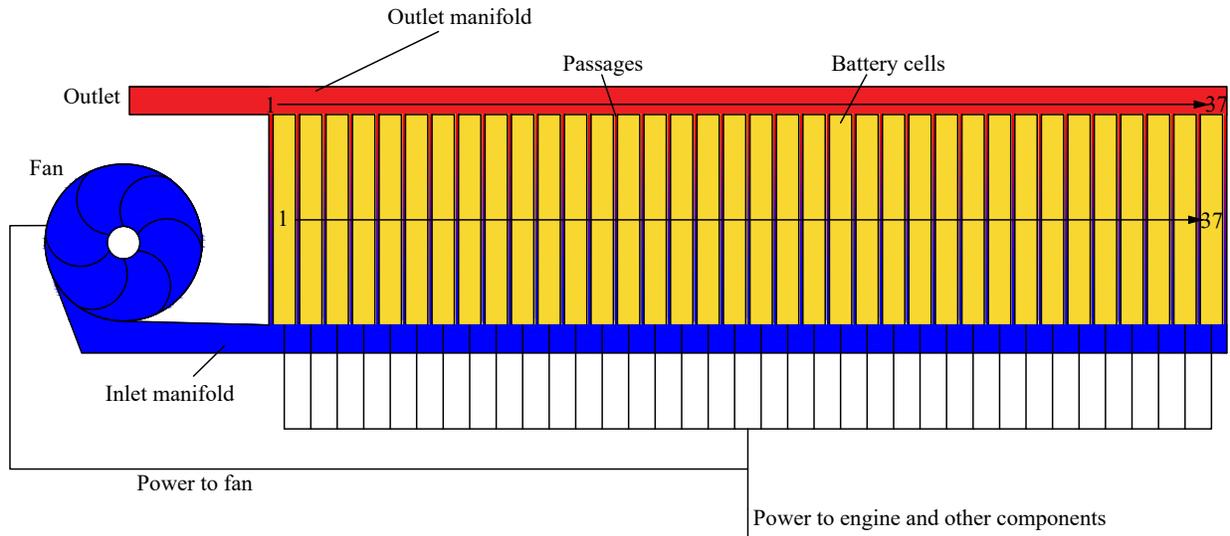


Figure 1. A typical air-based BTMS scheme.

which is described by Eq. (1).

$$P_{\text{Fan}} = \Delta p \dot{V} / \eta \quad (1)$$

where η is the efficiency of the fan. The equation shows that, for a given power, high pressure air by the fan means a low volume flow rate. The heat generated by a battery cell can be calculated by Eq. (2).¹⁷

$$q = I(U_o - U_c) = I^2 R_i \quad (2)$$

where q is the heat generated by battery cell; I is the current flowing through the battery cell; U_o is the open circuit voltage; U_c is the cell voltage; and R_i is the direct current internal resistance of the battery cell. Equation 2 shows that if the power of the fan increases, the heat generated by the battery cell also increases. In this study, the heat is converted to the heat flux on the battery cell surface, as shown in Eq. (3).

$$\dot{q} = q/A \quad (3)$$

where \dot{q} is the heat flux; A is the heat exchange area between the air and battery cell. The heat is taken away by the cooling air. When the battery cell is at a heat balance, the heat can be also described by Eq. (4).

$$q = \rho \dot{V} c_p (T_{\text{out}} - T_{\text{in}}) \quad (4)$$

where ρ is the density of air, and $\rho \dot{V}$ represents the mass flow rate of the air; c_p is a specific heat capacity; T_{out} and T_{in} are the air temperatures at the passage inlet and outlet, respectively.

The heat exchange between the battery cell and the air is calculated by Eq. (5)

$$q = h(T_{\text{cell}} - T_{\text{air}}) \quad (5)$$

where h is the heat transfer coefficient; T_{cell} is the battery cell temperature; T_{air} is the cooling air temperature. It is seen from Eqs. (4) and (5) that, the heat transfer significantly depends on the mass flow rate of the cooling air.

III. BTMS Modeling and Optimization

The BTMS is a complex engineering system that consists of a number of sub-models under different disciplines. For example, the layout of the battery cells affects the heat transfer; the air supply system

consumes battery power besides the cooling function. To successfully and effectively optimize the BTMS, a multidisciplinary design optimization (MDO) model of BTMS is developed.

In addition, building a high-fidelity physics-based model is also a challenging task due to the complexity of fluid flow and heat transfer. In the BTMS, the mass flow rates in different passages are different, which results in the temperature difference among battery cells. Due to the different air temperatures at the passage inlet and outlet, the temperature distribution on the battery cell is uneven as well. A high-fidelity CFD model of an air-based BTMS is developed to simulate the performance of the BTMS. This high-fidelity CFD model is accurate for simulation. However, we are not able to optimize the BTMS directly based on the high-fidelity CFD model due to its expensive computational time. To improve the optimization efficiency, surrogate models are developed to represent the key performance metrics as functions of design variables. The developed surrogate models are integrated into a MDO architecture and used in the MDO process.

A. CFD Model

In this study, the commercial software ANSYS 17.0 is utilized. The realistic $k - \varepsilon$ turbulence model and the SIMPLEC method are used to solve the simulations based on the steady and pressure-based solver. The second order upwind is used to disperse the pressure, density, and momentum equations. The thermal radiation transfer is assumed to be negligible in the work. Figure 2 shows the grid model. The air inlet is specified as the mass flow rate boundary condition, whereas the outlet is set as the pressure outlet boundary condition. The surface of the battery cell is set as the heat flux condition. The adiabatic and no-slip wall boundary condition is applied to the walls of the battery module.

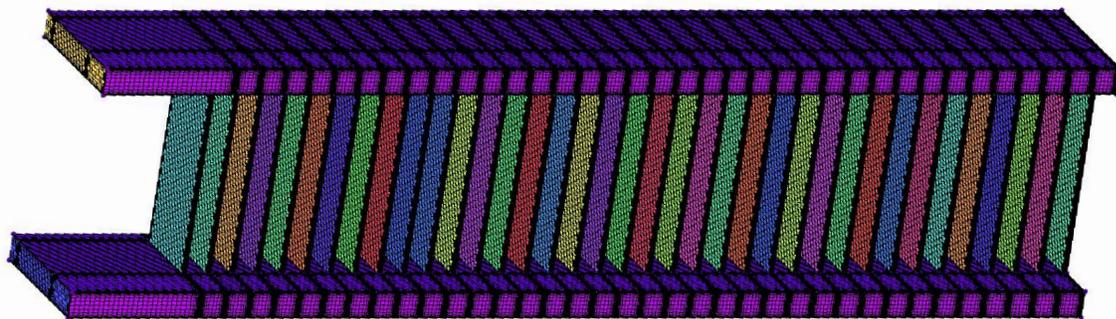


Figure 2. Grid model

In the simulations, three key design parameters are selected, including the mass flow rate of the cooling air (\dot{m}), the heat flux from the battery cell to the air (\dot{q}), and the passage spacing size (b). A design of experiments is performed, and each parameter is divided into 5 levels, as shown in Table 1. The full-factorial design is used in the design of experiments. Two outputs, including the pressure drop (Δp) and the maximum temperature difference (ΔT), are selected to evaluate the performance of the BTMS. Typical distributions of the total pressure and total temperature are shown in Fig. 3, where \dot{m} , \dot{q} , and b are 0.0225 kg/s, 245 W/m² and 3.0 mm, respectively. It is observed from the first passage to the last passage, the pressure difference between the passage inlet and outlet decreases, while the temperature increases.

Table 1. A design of experiments for the three key parameters

Parameters	Level 1	Level 2	Level 3	Level 4	Level 5
\dot{m} (kg/s)	0.0175	0.0200	0.0225	0.0250	0.0275
\dot{q} (W/m ²)	220	245	275	295	320
b (mm)	2.0	2.5	3.0	3.5	4.0

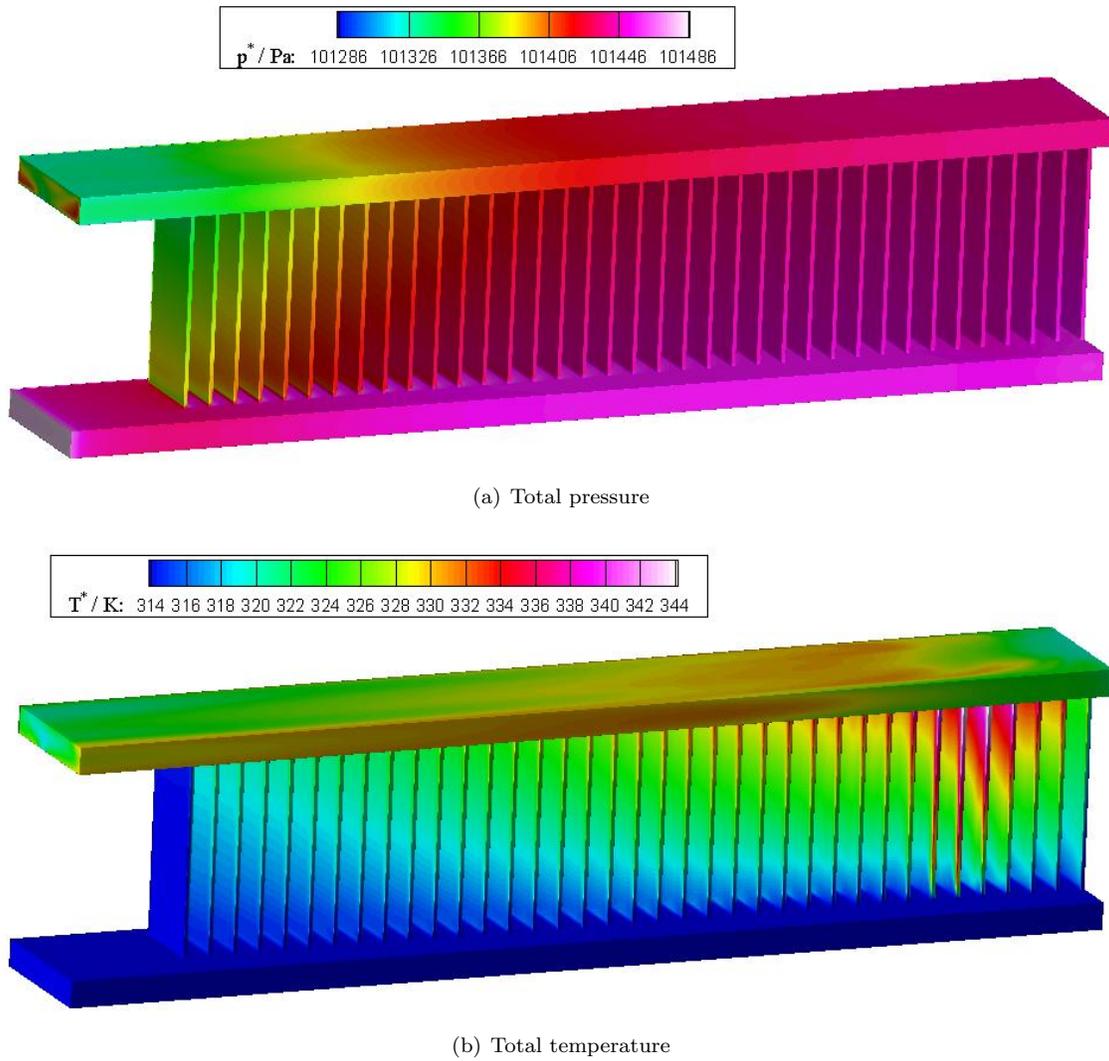


Figure 3. Distributions of the total pressure and the total temperature for a typical case

B. Surrogate Model

Surrogate modeling consists of three major components: design of experiments (DoE), surrogate model construction, and accuracy evaluation. A variety of DoE sampling methods have been available in the literature, such as the latin hypercube sampling,¹⁸ orthogonal array design,¹⁹ uniform design,²⁰ full-factorial design, central composite design,²¹ and so on. There also exist a number of surrogate modeling methods in the literature, such as the response surface method, Kriging,²² radial basis functions,²³ and support vector regression.²⁴ Kriging is adopted in this study to build surrogate models that represent the BTMS performance metrics as functions of design parameters. Figure 4 shows two approximated Kriging models based on the CFD simulations. The two functions are described as:

$$\Delta T = f(\dot{m}, \dot{q}, b) \quad (6)$$

$$\Delta p = f(\dot{m}, \dot{q}, b) \quad (7)$$

In order to assess the quality of the established surrogate models, the q -fold strategy is used to calculate the cross-validation error in this paper. The q -fold strategy usually has a lower computational cost, especially when the number of subsets is relative small. By dividing the data into q subsets randomly and removing

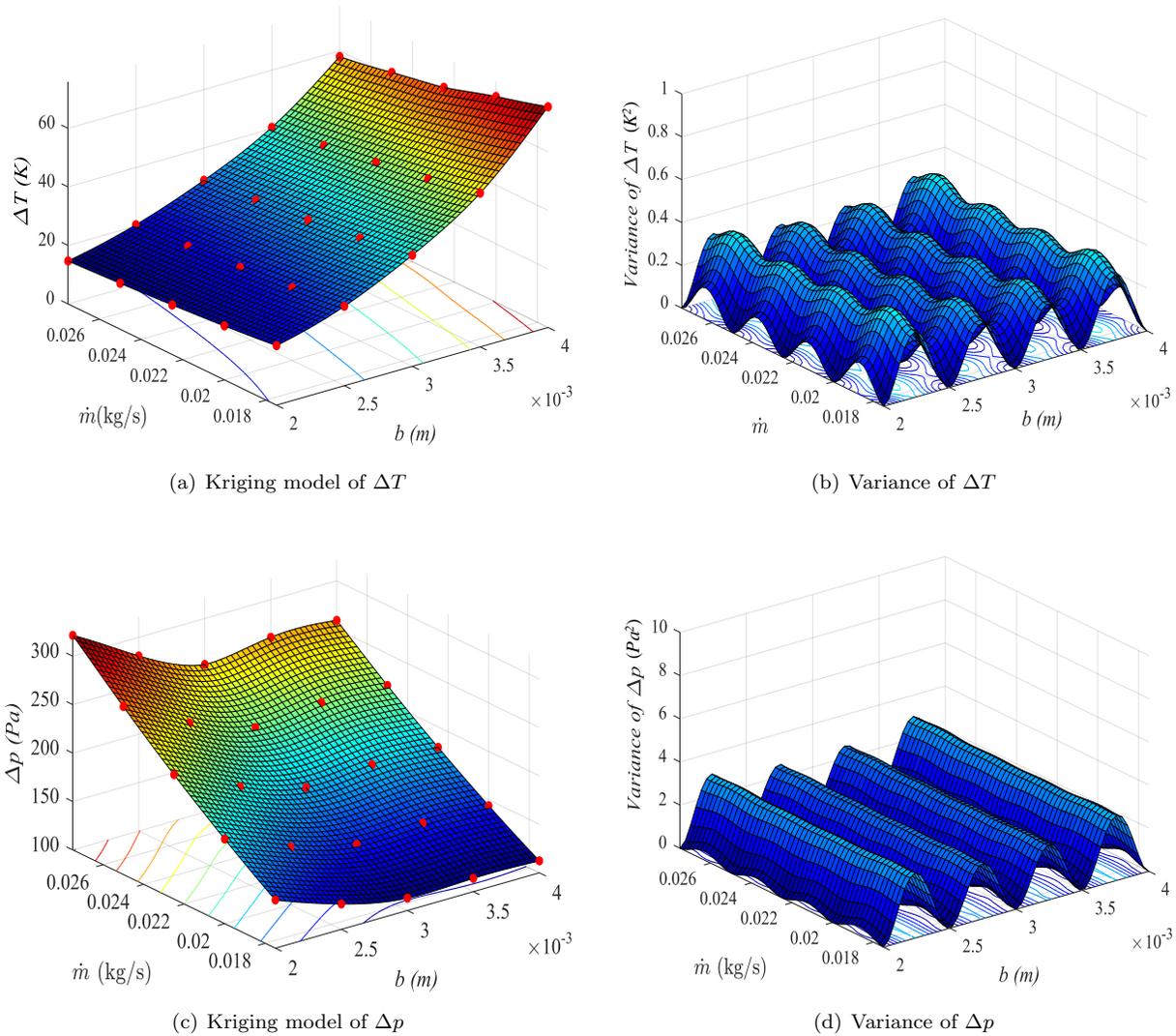


Figure 4. Surrogate models of the temperature difference ΔT and the pressure drop Δp .

each of the subsets in sequence, the cross-validation can be obtained as follows:

$$\text{PRESS}_{\text{SE}} = \frac{1}{n} \sum_{i=1}^n \left[y^{(i)} - \hat{f}^{-\zeta(i)}(x^{(i)}) \right]^2 \quad (8)$$

where ζ denotes the mapping: $1, \dots, n \rightarrow 1, \dots, q$, describing the allocation of the n training points to one of the q ; $\hat{f}^{-\zeta(i)}$ denotes the fitted model by removing the subset $-\zeta(i)$. As recommended by Jerome et al.,²⁵ q is set to be 5 in this paper. The 5-fold cross-validation errors of surrogates of ΔT and Δp are 4.85% and 2.47%, respectively.

As illustrated in Figs. 4(b) and 4(d), the variances of both the surrogate models built for the temperature difference and the pressure loss are reasonable, which ensures the reliability and feasibility of the following MDO process.

C. MDO Method

MDO has been widely used in many engineering elds.^{26–28} To overcome the computational challenge and find efficient optimization algorithms, a number of MDO architectures have been proposed in the literature.²⁹ MDO architectures can be divided into two main categories: single-level formulations (e.g., All-At-Once,³⁰

Multi-Disciplinary Feasible,³¹ and Individual Discipline Feasible³²) and multi-level formulations (e.g., Collaborative Optimization,³³ Concurrent Subspace Optimization,³⁴ and Bi-Level Integrated Synthesis System³⁵). The Multi-Disciplinary Feasible (MDF) architecture is adopted in the paper to solve the BTMS problem.

To successfully conduct the MDO of BTMS, the parameter coupling analysis between different disciplines needs to be done first. In this paper, the BTMS is decoupled into three sub-disciplines, namely the battery thermodynamics, fluid dynamics (air), and structure. Each of the sub-discipline has its own model however with different parameters coupled with each other. For example, the two state design variables of pressure drop and mass flow rate belong to thermodynamics and fluid dynamics, respectively. In addition, these two state design variables are coupled with each other through the CFD based surrogate models.

$$\dot{q} = \xi \frac{\Delta p \dot{m}}{36A\rho\eta} + \dot{q}_v \quad (9)$$

where η is the scale coefficient of the heat generation of the fan, and \dot{q}_v is the heat generation due to vehicle operations. Figure 5 illustrates the coupling effects and data exchange between different disciplines of the BTMS.

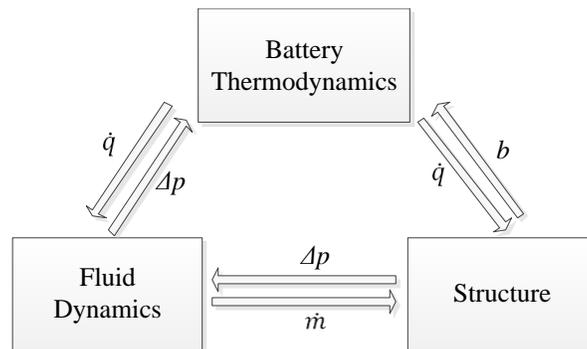


Figure 5. The data exchange between different sub-disciplines of the BTMS

In practice, there are different objectives in the design of BTMS, such as minimizing the battery size, the fan power, and the temperature difference between battery cells. In this paper, three parameters are optimized, including the mass flow rate, heat flux, and passage spacing size. This multi-objective optimization problem can be transformed into a single objective optimization problem using a weighted approach, as given by

$$f = \Delta T^{\lambda_1} P_{\text{Fan}}^{\lambda_2} b^{\lambda_3} \quad (10)$$

where λ is the weight. In this paper, the weights are set as $\lambda_1=0.8$, $\lambda_2=0.1$, and $\lambda_3=0.1$.

Based on the physical structure and performance limitation of the battery used in EVs, the upper and lower bounds of the three design variables are selected, as shown in Table 2.

Table 2. Upper and lower bounds of design variables

Design parameters	Lower bound	Upper bound
\dot{m} (kg/s)	0.0175	0.0275
\dot{q} (W/m ²)	220	320
b (mm)	2.0	4.0

The constraints in the optimization mainly include the battery performance metrics associated with the battery temperature, the air, and the structure. These constraints are summarized in Eq. 11.

$$\begin{cases} g_1 = \Delta T - 25 \\ g_2 = P_{\text{Fan}} - 3.5 \\ g_3 = \Delta p - 320 \\ g_4 = 110 - \Delta p \end{cases} \quad (11)$$

The overall MDO model of the BTMS is given by:

$$\begin{cases} \text{find } (\dot{m}, \dot{q}, b) \\ \text{min } f = \Delta T^{\lambda_1} P_{\text{Fan}}^{\lambda_2} b^{\lambda_3} \\ \text{s.t. } g_i \leq 0 \ (i = 1, 2, 3, 4) \\ \Delta p = f(\dot{m}, \dot{q}, b) \\ \dot{q} = \xi \frac{\Delta p \dot{m}}{36 A \rho \eta} + \dot{q}_v \end{cases} \quad (12)$$

The MDF, one of the most widely used MDO architectures, is chosen as the optimization algorithm to conduct the MDO process in this paper.²⁹ As shown in Fig. 6, the system optimizer distributes the design variables into the multiple disciplinary analysis (MDA) module, an indispensable analyzer to realize the interdisciplinary consistency. In the MDA module, each individual discipline receives the system design variables from the system optimizer, and the coupled design variables from other disciplines. The MDA analyzer transfers the state variables to the system optimizer to evaluate the constraints and objectives.

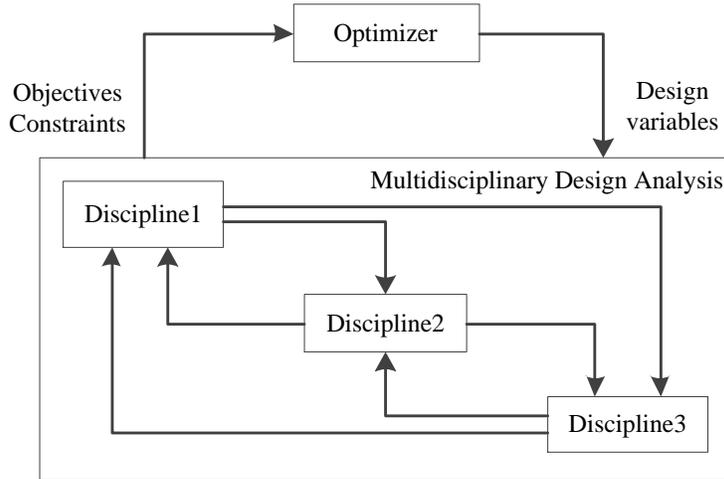


Figure 6. A typical Multi-Disciplinary Feasible architecture

IV. Results and Discussion

The optimization history of the objective is shown in Fig. 7(a), where the convergence curve approaches to the optimal solution at the 13th iteration, with the objective value of 7.73. The initial values of \dot{m}_0 and b_0 are set respectively as 0.025 kg/s and 3 mm. The parameter of \dot{q}_0 is a state design variable coupled with Δp , which can be obtained in every iteration during the MDO process. Thus, the initial heat flux \dot{q}_0 is obtained as 250.81 W/m² by the MDA analysis, and \dot{q}_v is set to be 250 W/m². As illustrated in Fig. 7, the optimal temperature difference ΔT is 23.38 K, the optimal fan power is 3.47 W, and the optimal channel size is 2.50 mm. The consistency of the MDF architecture is illustrated in Fig. 7(b), from which the consistency sufficiently decreases to a reasonable range. Thus, the optimization results of this MDO process are credible.

Table 3 compares the optimization results to the initiate values of the design variables and optimization objectives. The mass flow rate decreases from 0.025 kg/s to 0.019 kg/s by 24.00%, the heat flux decreases from 250.81 W/m² to 250.61 W/m² by 0.08%, and the passage spacing size decreases from 3 mm to 2.5 mm by 16.67%. The temperature difference decreases significantly by 17.91% from 28.48 K to 23.38 K. According to the surrogate model in Fig. 4(a), the decrease of heat flux and mass flow rate enables the reduction of temperature difference. The power consumed by the fan is reduced from 6.85 W to 3.47 by 49.34%. This remarkable decreased fan power is mainly contributed by the pressure drop reduction from 212.46 Pa to 143.71 Pa. Also the passage spacing size is reduced by 16.67%, indicating that the length of battery pack is shortened by 18.5 mm.

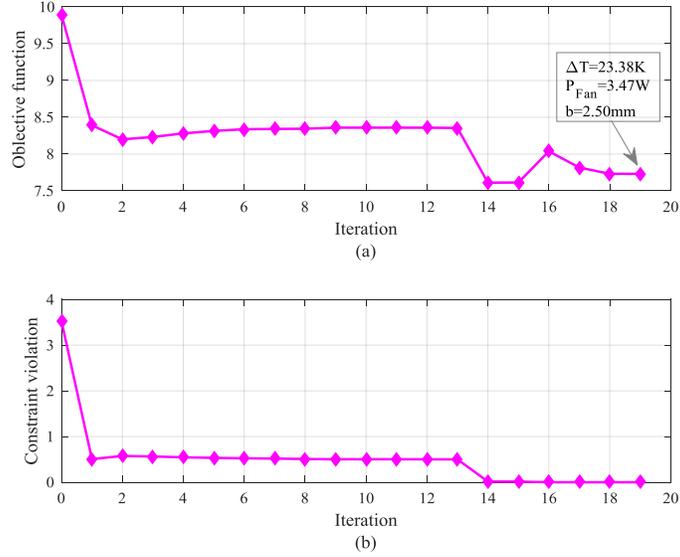


Figure 7. Optimization history of the objective and consistency

Table 3. MDO results of the BTMS

	Parameters	Initial value	Range	Optimal value	Improvement
Design variables	\dot{m} (kg/s)	0.025	[0.0175,0.0275]	0.019	24.00%
	\dot{q} (W/m ²)	250.81	[0,+∞]	250.61	0.08%
	b (mm)	3.00	[2,4]	2.50	16.67%
Objectives	ΔT (K)	28.48	Min	23.38	17.91%
	P_{Fan} (W)	6.85	Min	3.47	49.34%
	Δp (Pa)	3.00	Min	2.50	16.67%

V. Conclusion

A surrogate-based multidisciplinary design optimization (MDO) of an air-based battery thermal management system (BTMS) was developed in this paper. Three major sub-systems of the BTMS were modeled in the MDO model, including the air supply system, the battery cell, and the structure of the battery module. Design parameters of mass flow rate, heat flux, and passage spacing size were optimized, and the MDO objectives were to minimize the battery cells temperature difference, fan power, and battery size. To perform optimization based on the high-fidelity CFD models, Kriging surrogate models were developed. The optimization results showed that the thermal performance of the battery module was significantly improved by reducing the temperature difference, fan power, and battery size. In the future, an experimental platform will be developed to validate the simulations and optimization of the air-based BTMS.

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