# Distributed Solar Energy Sharing within Connected Communities: A Coalition Game Approach

Li He and Jie Zhang
The University of Texas at Dallas
Richardson, TX 75080, USA
Email: jiezhang@utdallas.edu

***Abstract*—As penetration of distributed energy resources such as rooftop photovoltaic (PV) increases, during certain hours of the day, power supplied by distributed generators is anticipated to exceed local consumption needs. This creates the potential to send power flows in the "reverse direction", which may create a technical challenge for the grid. This paper proposes to examine how the transactive energy might help solve this challenge and also disrupt electricity markets. Specifically, we model an environment in which owners of distributed PV can sell their excess generation to their neighbors through a virtual community exchange. The PV energy sharing is modeled as a cooperative game, which also considers the demand response (DR) of energy trading prosumers. A simplified reward allocation mechanism is developed based on the Shapley value and marginal contributions to allocate the coalition revenue. The objective of the coalition game among multi-prosumers is to maximize the total welfare of these prosumers increase by joining the coalition. A case study with 50 houses (all having distributed PV) in Texas shows that the coalition with both PV sharing and DR could improve cost savings for all prosumers.**

***Index Terms*—energy sharing, distributed PV, coalition game, demand response, transactive energy**

## I. INTRODUCTION

End users in power systems are undergoing a fundamental transition, from traditional passive "consumers" to active "prosumers", which allows them to actively manage their consumption and production [1]. The electric power grid was originally designed for power flow in one direction – from central generators to high voltage transmission networks, to distribution substations, to medium voltage distribution feeders and finally, through distribution transformers, to individual consumer loads. As penetration of distributed energy resources (DERs) such as rooftop photovoltaic (PV) increases, during certain hours of the day, power supplied by distributed generators is anticipated to exceed local consumption needs. This creates the potential to send power flows in the "reverse direction", from individual prosumers through the feeders back to the distribution substation and possibly into the transmission system, which may create a technical challenge because the distribution grid, in its present state, is not equipped to manage a large amount of reverse power flows, and upgrades that could pose significant challenges to the safety and reliability of power system operations.

California is already struggling with excess solar generation, which has driven wholesale grid prices down to zero on 178 days during 2016 [2]. Markets have recently began exploring other applications for excess PV generation that would curb the amount of reverse flows being fed into the grid. These alternative schemes derive from the "*transactive energy*", which has disrupted several economic sectors over the past decade by introducing new business models, marketing opportunities, and options for consumers and capital owners. *Transactive energy* [3] was defined as "a system of economic and control mechanisms that allows the dynamic balance of supply and demand across the entire electrical infrastructure using value as a key operational parameter". The price signal is transmitted among multi-prosumers in an energy sharing market, which allows the prosumers to react to different prices, return information back to the market, and automatically reschedule the their loads.

During different time periods, one prosumer can act as either a supplier or consumer, and all prosumers are equipotent participants in energy trading. Peer-to-Peer (P2P) energy trading [4] offers new opportunity to account for heterogeneous preferences of multi-prosumers. Encouraging energy trading among connected communities may also help prosumers make more profits comparing with trading with retailers or utility grid directly. This novel trading mode will potentially allow communities to reduce their reliance on the central grid and increase their resilience and ability to participate in DR or curtailment [5], achieve better generation-load balance, and prevent local building outages [6], which has shown to be a feasible way to reduce negative impacts on the utility grid [7].

This research is proposed to examine how the sharing economy might disrupt electricity markets. Specifically, we model an environment in which owners of small-scale distributed PV generation in addition to consuming their own generation and sending excess PV back into the grid, are given an opportunity to sell their excess generation to their neighbors through a virtual community exchange. The remaining of this paper is organized as follows: Section II gives the brief descriptions of game-based energy sharing. The developed PV sharing methodology and algorithm of the coalition game are discussed in Section III. Results of a case study with 50 prosumers are shown in Section IV and Section V gives the conclusion.

## II. GAME-BASED ENERGY SHARING

Game theory has been widely applied in P2P energy trading since it is effective in dealing with complicated interaction among energy suppliers and consumers. Several approaches have been proposed in the literature for P2P energy trading, such as peer-peer non-cooperative game [8], [9], leader-follower non-cooperative game [5], [7], and cooperative game [10]–[12]. A non-cooperative game offers insights into economic situations that involve multi-individuals who have different optimization goals or preferences, which can accurately simulate practical trading in the electricity market. However, there is no guarantee of the existence and uniqueness of the equilibrium, and the calculation could be complicated by using iterative algorithm to reach Nash or Stackelberg equilibrium.

Different from non-cooperative game, a cooperative game seeks to maximize the total welfare of the coalition. For the grand coalition, the profit allocation issue is of high importance. Shapley value [10] is one way to achieve a fair distribution of revenues in a coalition, however it is computationally complex and time-consuming when the number of participants gets larger. To address this challenge, Lee et al. [11] introduced an asymptotic Shapley value, which can directly decide the electricity price based on the number of participants, power supply, and load demand. Liu et al. [12] developed a two-level Shapley value to allocate the coalition profit. This paper formulates a coalition energy sharing scheme among a number of prosumers, seeking to explore if the total welfare of these prosumers increases by forming a coalition. Based on the contribution of each coalition member, a simplified reward allocation scheme is proposed to allocate the profit, which is readily to be adopted in practical scenarios.

## III. Pricing and Optimization of PV Prosumers

### A. Coalition Game

A coalition game is characterized by sets of multi-players $\mathcal{N}$ and a value function $v$ that assesses the profit of the coalition. The participated prosumers can form a union, in contrast to an independent model, the overall cost of the coalition is expected to decrease, which is defined as:

$$v(\mathcal{N}) = C_i(\mathcal{N}) - C_c(\mathcal{N}) \tag{1}$$

where $C_i(\mathcal{N})$ and $C_c(\mathcal{N})$ are the total cost function of all prosumers in an independent model and a cooperation model, respectively.

$$\mathrm{C}_i(\mathcal{N}) = \sum_{i\in\mathcal{N}} C_i(i) \tag{2}$$

In the independent model, the objective function of prosumer $i$ is given by:

$$\mathrm{C}_i(i) = p_b \cdot (x_n - P_{pv})^+ + p_s \cdot (P_{pv} - x_n)^+ \tag{3}$$

where we define $(x)^+ = max(x, 0)$. The parameters $p_b$ and $p_s$ denote the buying and selling prices from/to the utility grid; $x_n$ and $P_{pv}$ denote load and PV generation of prosumer $i$, respectively. In order to encourage the self-consumption of renewable energy, selling price is lower than buying price and thus $p_s < p_b$. Then the cost of individual prosumer can be divided into two parts: (i) cost of purchasing power from the utility grid, and (ii) revenue of selling power to the utility grid.

Due to the variation in PV generation, load, and DR, the roles of prosumers can be dynamically changed between buyers ($\mathcal{N}_b$) and sellers ($\mathcal{N}_s$), and $\mathcal{N} = \mathcal{N}_b + \mathcal{N}_s$. In the cooperative model, the cost function is defined as:

$$\mathrm{C}_c(\mathcal{N}) = \sum_{i\in\mathcal{N}_b} c_n^+ + \sum_{i\in\mathcal{N}_s} c_n^- \tag{4}$$

$$c_n^+ = p_b(x_n - P_{pv} - x_{in}) + p_{in}x_{in} \tag{5}$$

$$c_n^- = p_s(P_{pv} - x_n - x_{in}) - p_{in}x_{in} \tag{6}$$

where $\sum_{i\in\mathcal{N}_b} c_n^+$ and $\sum_{i\in\mathcal{N}_s} c_n^-$ stand for the cost of all buyers and sellers, respectively. The parameter $x_{in}$ denotes the shared energy from/to other prosumers and $p_{in}$ is the sharing price.

$$\mathrm{C}_c(\mathcal{N}) = p_b \sum_{i=1}^{\mathcal{N}} (x_n - P_{pv})^+ + p_s \sum_{i=1}^{\mathcal{N}} (P_{pv} - x_n)^+ \tag{7}$$

By substituting (5) and (6) into (4), we get (7) and find that the internal sharing price and energy sharing don't affect the value function, because the transactions between prosumers don't result in monetary change for the coalition as a whole.

In a coalition game, when the value function is superadditive, forming a grand coalition is optimal for maximizing the total revenue of the participants [11]. In this paper, since the sharing price $p_{in}$ is always more competitive than the utility price (which will be introduced in the next subsection), the value function is always superadditive. Thus forming a coalition, for all prosumers, no matter what their roles are, is optimal for maximizing their revenues or minimizing costs. Thus it is always beneficial for every prosumer to join the energy sharing coalition.

### B. Sharing Price

In energy sharing, the internal sharing price can be used as a way to distribute the revenue. For example, if $p_{in}$ is too high, then more revenue of the coalition will be attributed to the sellers; similarly, buyers will benefit more if $p_{in}$ is too low. Following the Theorem 3 in [11], the value of $p_{in}$ is directly defined as follows:

1) if $\eta < 1$, $p_{in} = p_s$
2) if $\eta > 1$, $p_{in} = p_b$
3) if $\eta = 1$, $p_{in} = \frac{1}{2}(p_s + p_b)$

The $\eta$ denotes the ratio of the total demand to the total PV generation of the coalition. This pricing mechanism shows that when the total amount of PV generated by prosumers doesn't match the total load, the sharing price will be set as two extreme points. For example, when the PV generation is less than the demand ($\eta > 1$), the price converges to $p_b$ and the buyers become price taker. While when PV is over-supplied ($\eta < 1$), sellers lose their negotiation power and the sharing price becomes $p_s$ which solely benefits the buyers. Here we set $p_{in} = \frac{1}{2}(p_s + p_b)$ when $\eta = 1$ to ensure the benefits of all prosumers when the PV generation matches the load in the coalition.

### C. Simplified Reward Allocation

The Shapley value can be regarded as a measure of the contribution made by every individual to the coalition.

$$\phi_i\left(v\right) = \sum_{\mathcal{S}\subset\mathcal{N}\setminus i} \frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!}\left[v\left(\mathcal{S}\cup i\right)-v\left(\mathcal{S}\right)\right] \quad (8)$$

where $\phi_i(v)$ is the Shapley value of prosumer $i$; $\mathcal{N}$ and $\mathcal{S}$ denote the number of players in grand coalition and sub-coalition except prosumer $i$, respectively; the term $v\left(\mathcal{S}\cup i\right)-v\left(\mathcal{S}\right)$ denotes the marginal contribution of player $i$ and the item $\frac{|\mathcal{S}|!(|\mathcal{N}|-|\mathcal{S}|-1)!}{|\mathcal{N}|!}$ is a weighting factor.

Although the Shapley value provides a fair distribution among the players, it is challenging to calculate the Shapley value when a large number of participants join the coalition. In this paper, we develop a simplified reward allocation strategy based on each prosumer's individual contribution to the coalition. For prosumer $i$, the contribution to the coalition $C_{cont.i}$ is calculated by:

$$C_{cont.i} = C_c(\mathcal{N}\setminus i) + C_i(i) - C_c(\mathcal{N}) \quad (9)$$

$$\lambda_i = \frac{C_{cont.i}}{\sum_{i=1}^{\mathcal{N}} C_{cont.i}} \quad (10)$$

$$Cost_i = C_i(i) - \lambda_i v(\mathcal{N}) \quad (11)$$

Equation (9) denotes the contribution of prosumer $i$ to the coalition. The parameter $\lambda_i$ is a weighting factor, or an allocation ratio. So the final cost of every prosumer $Cost_i$ is given in (11), and $v(\mathcal{N})$ is the value function as defined in (1).

### D. Demand Response (DR)

In smart grid, we assume every prosumer is rational and will respond to different electricity prices to maximize the profit or reduce the cost [13]. For example, when the electricity price is high, consumers can reduce their consumption or shift their loads to lower electricity price period, while prosumers can sell excess energy at a higher price. In a DR program, every prosumer is assumed to have a certain portion of shiftable loads such as washers, dishwashers, electric vehicles, air conditioners, and water heater, which allows prosumers to choose the time for using their devices according to the time-of-use (ToU) price. In this paper, the energy sharing strategy is constructed together with DR to further reduce the cost of electricity use.

It is assumed that with PV sharing, the prosumers will first use the PV power generated by their panels, and then trade energy with the utility grid or other prosumers. Due to the price incentive, prosumers may change their energy consumption behavior, which makes their load curves to deviate from original ones. The flexible load should satisfy the following constraints:

$$\sum_{t=1}^{H} x_{ini.i} = \sum_{t=1}^{H} x_{opt.i} \quad (12)$$

$$x_{opt.imin} \leq x_{opt.i} \leq x_{opt.imax} \quad (13)$$

Where $H$ denotes the numbers of time slots, which is set to be 24 hours in a day. The terms $x_{opt.i}$ and $x_{opt.i}$ stand for the initial load and optimized load of prosumer $i$, and $x_{opt.imin}$ and $x_{opt.imax}$ stand for the minimum and maximum load of prosumer $i$, respectively, which are used to assure that the optimized load is between the baseload and the upper limit of supply capacity.

By considering DR together with energy sharing, we should substitute $x_{n.i}$ with $x_{opt.i}$ and recalculate aforementioned Eqs. (1)-(11). Then the overall framework of PV energy sharing with DR can be summarized as **Algorithm 1**.

## IV. Simulations & Results

This paper has applied the developed PV sharing method to a case study with 50 houses (all having distributed PV) in Texas [14]. The ToU tariff price is adopted from ref. [15]. The on-peak hours are (i) 3pm-8pm in the months of May through October, and (ii) 6am-8am and 3pm-8m in the months of November through April. All other hours are classified as off-peak hours. The on-peak and off-peak prices are 18.4 and 9.1 ¢/kWh, respectively, and the selling price ($p_s$) is set to be 7 ¢/kWh.

Figure 1 shows the load and PV output of the 50 houses of a typical summer day on August 29, 2016. It is shown from Fig. 1(a) that the load at each house varies significantly over the day, and the peak of the average load occurs at around 17:00, as shown by the red line. To simplify the computation, the power loss is neglected in this system since these prosumers' houses are within the same neighborhood. The simulation is run in Matlab R2017b.

Figure 2 shows the aggregated load of a typical day under three different scenarios: (i) original load without PV sharing and DR; (ii) load with DR however without PV sharing; (iii) load with both PV sharing and DR. The PV generation is also shown in the figure. The percentage of the shiftable load

**Algorithm 1:** Framework of PV energy sharing with demand response in a coalition game

```
for each prosumer i ∈ N do
    Collect x_{ini,i}, P_{pv}, [x_{opt,imin}, x_{opt,imax}], calculate
    C_{i.n} according to (3), then report to central data
    processor;
end for
Central data processor collects data from all prosumers
and calculates C_c(N), v(N) according to (7) and
(1)-(2);
for  i = 1 to N  do
    Central data processor calculates C_c(N \ i), λ_i, and
    x_opt according to (9)-(13);
end for
for each prosumer i ∈ N do
    Reschedule consumption according to x_opt and get
    final cost Cost_i according to (11);
end for
```

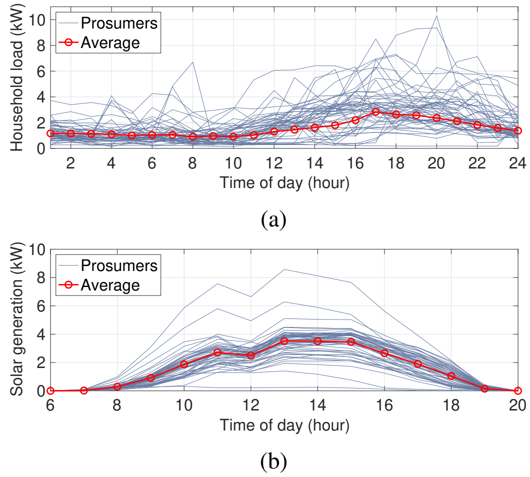


Fig. 1: Load and PV generation of a typical summer day on August 29, 2016: (a) aggregated and individual load; (b) aggregated and individual PV power.

is simulated at 5%, 15%, and 25% for all cases. With the increase of flexible load, the peak load at 16:00 - 21:00 is reduced and shifted to the off-peak period of 0:00 to 8:00. In addition, prosumers consume more electricity at 10:00 - 15:00 under the scenarios of individual DR and integrated PV sharing & DR, compared to the scenario without DR & PV sharing. This is because that the PV power generation is sufficient at 10:00 - 15:00 and the internal PV sharing price is cheaper than the ToU tariff price. Overall the flexible load is shifted from on-peak period to off-peak or period when PV is sufficient.

TABLE I: Comparing total costs of 50 prosumers under different scenarios

| Flexible load (%) | Original cost ($) | Individual DR ($) | Coalition DR ($) | $Imp_1$ | $Imp_2$ |
|---|---|---|---|---|---|
| 0% | 157.83 | 157.83 | 151.26 | 0 | 4.16% |
| 5% | 157.83 | 153.99 | 147.94 | 6.27% | 3.93% |
| 15% | 157.83 | 146.43 | 141.41 | 10.40% | 3.43% |
| 25% | 157.83 | 139.12 | 135.14 | 14.38% | 2.86% |

*$Imp_1$ stands for the cost reduction percentage of the individual DR compared with original cost, and $Imp_2$ stands for the cost reduction percentage of the coalition DR compared with the individual DR.

The results of prosumers' costs under different scenarios are summarized in TABLE I. It is seen from the table that the proposed integrated PV sharing & DR strategy has shown improved cost savings compared to both the original case (without PV sharing and DR) and the independent DR case. Although by responding to tariff price independently, prosumers can reduce their costs through DR, joining in the coalition can further improve cost savings. By comparing the case of integrated PV sharing & DR with the independent DR case, it is observed that the group cost savings from PV sharing decreases with the increase of flexible load, since prosumers can save more by shifting their own flexible loads.

The group revenue should be allocated to each prosumer according to the simplified reward allocation defined in Section

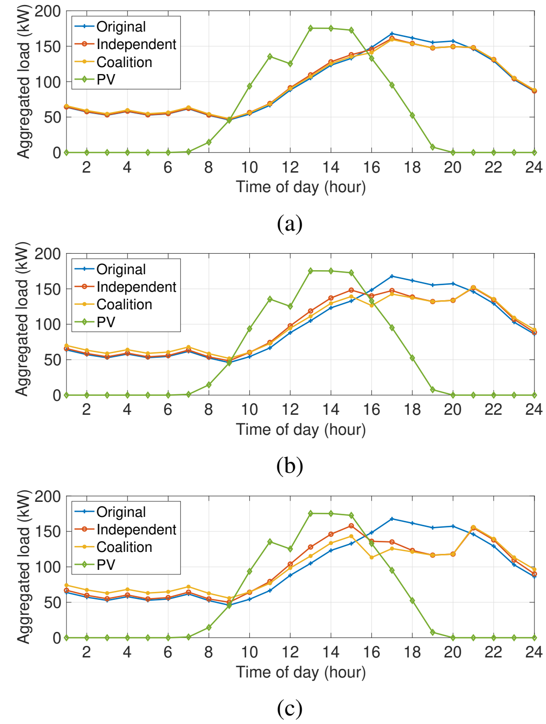


Fig. 2: Aggregated load and PV power with (a) 5%, (b) 15%, and (c) 25% flexible load.

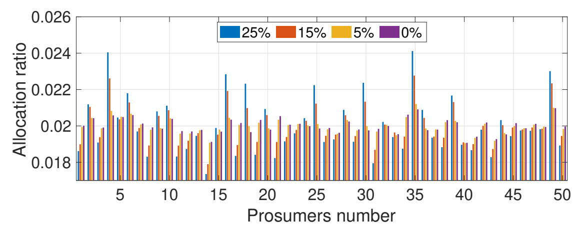


Fig. 3: The allocation ratio of the group revenue with different flexible load percentages.

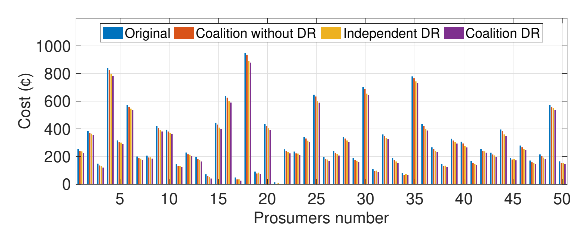


Fig. 4: The costs of 50 prosumers with 15% flexible load on August 29, 2016.

III-C and the result is illustrated in Fig. 3. It is seen that each prosumer's contribution to the coalition varies under the four different flexible load scenarios, due to the fact that prosumers' roles may change between sellers and buyers in different periods. For example, the prosumers can choose to sell surplus PV power to the buyers in the coalition, or change their electricity demand to consume surplus PV power. Figure

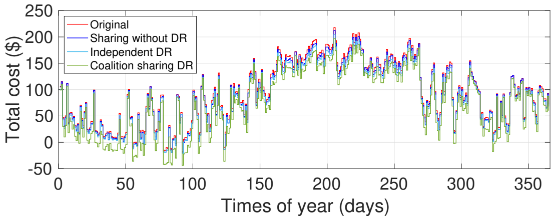


Fig. 5: Total costs of all prosumers throughout the whole year with 15% flexible load.

4 shows the final cost of all prosumers with 15% flexible load on August 29, 2016. It is observed that the coalition model has generated more cost savings than both the cases of independent DR optimization and PV sharing without DR. Since all players in the coalition make contribution and share the group revenue, no matter what their roles are, the coalition is stable and no player has the incentive to leave the coalition.

Figure. 5 shows the aggregated daily total cost of all prosumers over a year under the four scenarios: original, coalition sharing without DR, independent DR, and coalition DR. It is seen that the overall electricity costs in winter months are lower than those in summer months, due to the high summer cooling demand in Texas. In winter, there are more surplus PV to be shared in the coalition, leading to lower electricity bills for all prosumers. A few negative cost winter days are observed, which means the coalition can earn additional benefits by selling excess PV to the utility grid. It is seen that the total cost of the coalition sharing & DR is lower that of the independent DR, which means both buyers and sellers can obtain more monetary profits by participating in the coalition.

To evaluate the performance of the proposed sharing strategy under different ToU price scenarios, a case study with 12.4 ¢/kWh on-peak price is performed, which reduces the on- and off-peak price difference to 3.3 ¢/kWh. Table II shows the results in the same day with the new ToU prices, and it also shows the effectiveness of the coalition game. It is seen that the $Imp_1$ decreases when lowing the on-peak price, which indicates that prosumers' incentives of participating in DR decrease with lower on-peak price. However, prosumers can still achieve higher welfare by joining in the coalition since the $Imp_2$ increases by lowing the on-peak price.

TABLE II: Comparing total costs of 50 prosumers under new ToU prices (12.4 & 9.1 ¢/kWh for on-peak and off-peak hours, respectively)

| Flexible load (%) | Original cost ($) | Individual DR ($) | Coalition DR ($) | $Imp_1$ | $Imp_2$ |
|---|---|---|---|---|---|
| 0% | 125.93 | 125.93 | 119.40 | 0 | 5.19% |
| 5% | 125.93 | 124.26 | 118.29 | 1.33% | 4.80% |
| 15% | 125.93 | 120.97 | 116.12 | 3.94% | 4.01% |
| 25% | 125.93 | 117.78 | 114.06 | 6.47% | 3.16% |

## V. Conclusion

This paper developed a PV energy sharing framework based on cooperative game theory. A coalition game among multi-prosumers was formulated to simulate the PV energy trading behavior, and explore the total welfare of these prosumers increase by joining the coalition. Demand response (DR) of energy trading players is also modeled within the coalition game. The developed coalition with PV sharing and DR method was compared with three benchmarks: (i) conventional case without PV sharing and DR, (ii) coalition with PV sharing only, and (iii) independent DR without PV sharing. The results of a case study with 50 houses showed that: (i) the developed integrated PV sharing & DR strategy has shown improved cost savings compared to both the conventional case (without PV sharing and DR) and the independent DR case; (ii) the group cost savings from PV sharing decreases with the percentage increase of flexible load.

Future work will explore: (i) the impacts of solar and load forecasting on PV sharing, and (ii) the impacts of adding energy storage into the game.

## References

[1] Y. Parag and B. K. Sovacool, "Electricity market design for the prosumer era," *Nature Energy*, vol. 1, p. 16032, 2016.
[2] C. Sweet, "How california utilities are managing excess solar power," *The Wall Street Journal*, March 4, 2017.
[3] R. Melton, "Gridwise transactive energy framework," *Pacific Northwest National Lab.(PNNL), Richland, WA (US), Tech. Rep*, 2013.
[4] T. Morstyn, N. Farrell, S. J. Darby, and M. D. Mcculloch, "Using peer-to-peer energy-trading platforms to incentivize prosumers to form federated power plants," *Nature Energy*, vol. 3, no. 2, 2018.
[5] J. Lee, J. Guo, J. K. Choi, and M. Zukerman, "Distributed energy trading in microgrids: A game-theoretic model and its equilibrium analysis," *IEEE Transactions on Industrial Electronics*, vol. 62, pp. 3524–3533, June 2015.
[6] E. Mengelkamp, J. Gärttner, K. Rock, S. Kessler, L. Orsini, and C. Weinhardt, "Designing microgrid energy markets: A case study: The brooklyn microgrid," *Applied Energy*, vol. 210, pp. 870 – 880, 2018.
[7] N. Liu, X. Yu, C. Wang, C. Li, L. Ma, and J. Lei, "Energy-sharing model with price-based demand response for microgrids of peer-to-peer prosumers," *IEEE Transactions on Power Systems*, vol. 32, pp. 3569–3583, Sept 2017.
[8] B. Chai, J. Chen, Z. Yang, and Y. Zhang, "Demand response management with multiple utility companies: A two-level game approach," *IEEE Transactions on Smart Grid*, vol. 5, pp. 722–731, March 2014.
[9] Y. Wang, W. Saad, Z. Han, H. V. Poor, and T. Başar, "A game-theoretic approach to energy trading in the smart grid," *IEEE Transactions on Smart Grid*, vol. 5, pp. 1439–1450, May 2014.
[10] W. Saad, Z. Han, M. Debbah, A. Hjorungnes, and T. Basar, "Coalitional game theory for communication networks," *IEEE Signal Processing Magazine*, vol. 26, pp. 77–97, September 2009.
[11] W. Lee, L. Xiang, R. Schober, and V. W. S. Wong, "Direct electricity trading in smart grid: A coalitional game analysis," *IEEE Journal on Selected Areas in Communications*, vol. 32, pp. 1398–1411, July 2014.
[12] N. Liu, J. Wang, X. Yu, and L. Ma, "Hybrid energy sharing for smart building cluster with chp system and pv prosumers: A coalitional game approach," *IEEE Access*, vol. 6, pp. 34098–34108, 2018.
[13] P. Palensky and D. Dietrich, "Demand side management: Demand response, intelligent energy systems, and smart loads," *IEEE Transactions on Industrial Informatics*, vol. 7, pp. 381–388, Aug 2011.
[14] Pecan Street Inc. Dataport, 2018. https://dataport.cloud
[15] Denton County Electric Cooperative, "Tariff for electric service," 2018.