A Copula Enhanced Convolution for Uncertainty Aggregation

Binghui Li, Member, IEEE, Jie Zhang, Senior Member, IEEE
The University of Texas at Dallas
Richardson, TX 75080, USA
Email: {binghui.li, jiezhang}@utdallas.edu

Benjamin F. Hobbs, Fellow, IEEE
Johns Hopkins University
Baltimore, MD 21218, USA
Email: bhobbs@jhu.edu

Abstract—A promising approach to managing the uncertainty of renewables is probabilistic forecasting. However, a key challenge associated with the integration of probabilistic forecast into real-world applications is to estimate the distribution of the aggregation of several correlated variables given their individual distributions. This study presents a copula enhanced convolution technique that accounts for cross-variable correlations. The method approximates the distributions of the convolved variables with piecewise polynomial functions and the correlations are then discretized and factorized for better computation performance. The method is demonstrated by applying it to the convolution of two correlated random variables as well as multiple correlated variables. Our results indicate the copula enhancement effectively improves the results by reducing deviations from the actual distributions in the case of net load forecasting in California.

Index Terms—Probabilistic forecast, Convolution, Copula, Net load

I. INTRODUCTION

The benefit of adopting probabilistic forecast in power system scheduling and operating is being increasingly recognized in the light of increased penetration of renewable energy. By combining with frameworks of uncertainty analysis, such as scenario analysis, stochastic optimization, and robust optimization, probabilistic forecast has been widely adopted in the area of reserve determination [1], [2], unit commitment and economic dispatch [3], [4], energy storage sizing [5], and transmission capacity planning [6]. However, a key challenge associated with the usage of probabilistic forecast is to estimate the uncertainty range of an aggregated amount given the uncertainty range of all constituent components. Such examples include the aggregated output from a wind farm given forecasts of all individual wind turbines, or to estimate net load, which is typically a linear combination of total load and several renewable components, such as solar and wind power. The latter example is an important problem faced by the California Independent System Operator (CAISO), where the requirements of flexible ramping products and regulating reserves largely depend on estimating the uncertainty range of net load [7].

A fundamental approach to model and combine component uncertainties associated with a forecasted variable is to use probability distribution functions (PDFs), which can be constructed with a parametric approach or a non-parametric approach. The parametric approach represents the PDF with a closed-form function that is dictated by one or more parameters, while in the non-parametric approach such as quantile regression or kernel density estimation, the predictive PDFs are estimated at a finite number of representative points [8]. Therefore, a computationally efficient and mathematically rigorous method must be adopted to estimate the PDF of the aggregation of multiple random variables given their individual PDF. Typical approaches include convolution [9], [10], [2], Monte Carlo simulation [11], [12], and the cumulant method [13]. In [2] and [10], Etingov et al. manage to reduce the requirements for frequency regulation in CAISO by employing probabilistic forecasts, where convolution plays a pivotal role in bridging the gap between the uncertainty range of probabilistic forecasts of renewable sources and net load.

Convolution is a mathematical operation that integrates the product of two functions after one is reversed and shifted [14]. Conventionally, Fast Fourier Transforms (FFT) can be used to reduce the computation time of convolution [15]. However, the two convolved random variables are presumed to be independent, while real-world variables are usually correlated [12]. In particular, wind turbines or solar panels in a real-world power system are usually geographically close, which results in strong correlations in their power output. Neglecting such correlations may lead to systematic misrepresentation of their uncertainty, therefore place severe impact on power system security. In addition, previous studies have suggested that the power output from solar power and end-use electric demand present a stronger correlation and results in a higher capacity credit towards resource adequacy planning [16]. Therefore, to correctly represent the overall distribution of the aggregation of several correlated variables, an effective method must be implemented to account for the correlations across variables.

This study develops a numerical technique to calculate the PDF of the linear combination of two or more correlated random variables, given their individual distributions. Built on a technique for convolving piecewise polynomial functions [17], the cross-variable correlations are accounted for using copulas. In addition, the proposed approach can be generalized to the aggregation of any number of random variables with any type of distributions.

II. METHOD

A. Sum of random variables

Suppose that X and Y are two continuous random variables with PDFs $f_X, f_Y : \mathbb{R} \to \mathbb{R}_+$, and their joint PDF is denoted by $f_{XY}(x, y)$, then $X + Y$ is a continuous random variable whose PDF can be given by the following integral:
\[ f_{X+Y}(X+Y = z) = \int_{-\infty}^{\infty} f_{XY}(t, z-t) \, dt \]  

(1)

When \( X \) and \( Y \) are independent, the joint PDF \( f_{XY} = f_X \cdot f_Y \) and \( f_{X+Y} \) can be further written as the convolution (*) of \( f_X \) and \( f_Y \):

\[ f_{X+Y}(X+Y = z) = \int_{-\infty}^{\infty} f_X(z-y)f_Y(y) \, dy = f_X \ast f_Y \]  

(2)

B. Convolution of piecewise polynomial functions

Without knowing the explicit form of the convolved PDFs, we use piecewise polynomial functions to approximate them. The piecewise function should be able to approximate the original distribution with improved accuracy by refining the bin size. Without loss of generality, an \( I \)-segment piecewise polynomial function can be written in the following form:

\[ f(t) = f(t;i), \forall t \in [t_i, t_{i+1}], i \in \{1, \cdots, I\} \]  

(3a)

\[ f(t;i) = \sum_{m=1}^{M} \alpha_{im} t^{m-1} \]  

(3b)

where \( t_i, t_{i+1} \) represent the lower and upper bound of interval \( i \) and \( M - 1 \) denotes the highest order of \( f(t) \).

By approximating any distributions with piecewise polynomial functions, the results from convolution are still piecewise polynomial functions of higher orders, since integration of two polynomial functions still results in a polynomial function. However, due to discontinuity, the integral must be repeated for each possible combination of intervals of the two functions. Therefore, to avoid the repeated integral, Polge and Hays [17] give a single expression form of the piecewise polynomial function by using the Dirac delta function:

\[ f(t) = \sum_{i=1}^{I} \sum_{m=1}^{M} \alpha_{im} \delta^{-m}(t - t_i) \]  

(4)

where \( \delta^{-m}(t) \) represents the \( m \)th order integral of \( \delta(t) \), the Dirac delta function:

\[ \delta^{-m}(t - t_i) = \begin{cases} 0, & \text{if } t < t_i \\ \frac{(t-t_i)^{m-1}}{(m-1)!}, & \text{if } t \geq t_i \end{cases} \]

The detailed transformation between the per integral form in (3) and the delta function form in (4) can be found in [17]. In addition, [17] also gives the formula for convolution of two delta function integrals:

\[ \alpha_{im} \delta^{-m}(t - t_i) \ast \beta_{jn} \delta^{-n}(t - t_j) = \alpha_{im} \beta_{jn} \delta^{-m-n}(t - t_i - t_j) \]  

(5)

Therefore, the convolution integral of two delta functions reduces to elementary algebraic operations of two triplet elements:

\[ \begin{bmatrix} \alpha_{im} \\ t_i \\ -m \end{bmatrix} \ast \begin{bmatrix} \beta_{jn} \\ t_k \\ -n \end{bmatrix} = \begin{bmatrix} \alpha_{im} \beta_{jn} \\ t_i + t_k \\ -m - n \end{bmatrix} \]  

(6)

In summary, when \( f_X \) and \( f_Y \) in (1) are approximated by piecewise polynomial functions in the form of Dirac delta function in (4), the convolution integral is converted into a set of simple algebraic sums and products, which can be calculated efficiently.

C. Copula enhancement

A critical prerequisite for convolution is that the convolved components must be independent, however, correlations often exist across real-world random variables [12]. Therefore, copulas are employed to model the correlations in this analysis. According to Sklar’s theorem [18], any multivariate joint distribution can be written in terms of the marginal distributions of each component and a copula that describes the dependence structure between the components. Let \( F_X, F_Y \) denote the marginal cumulative density functions (CDFs) associated with \( X \) and \( Y \), the joint CDF \( F_{XY} \) becomes:

\[ F_{XY}(X \leq x, Y \leq y) = C(u, v) \]  

(7)

where, \( u = F_X(X \leq x) \), \( v = F_Y(Y \leq y) \) and the copula \( C(\cdot) : [0, 1]^2 \to \mathbb{R}_+ \) is a continuous, real-valued function. Therefore, the joint PDF in (1) can be transformed into:

\[ f_{XY} = \frac{\partial^2 F_{XY}}{\partial X \partial Y} = \frac{\partial^2 C}{\partial u \partial v} \cdot f_X \cdot f_Y \]  

(8)

Therefore, the joint PDF of \( X \) and \( Y \) can be simplified as

\[ f_{XY} = c(u, v) \cdot f_X \cdot f_Y \]  

(9)

where, function \( c(\cdot) : [0, 1]^2 \to \mathbb{R}_+ \) is given by:

\[ c(u, v) = \frac{\partial^2 C}{\partial u \partial v} \]  

(10)

With the above copula representation, the integral in (1) becomes:

\[ f_{X+Y}(z) = \int_{-\infty}^{\infty} c \cdot f_X(t) \cdot f_Y(z-t) \, dt \]  

(11)

Therefore, the convolution in (2) is multiplied by a copula function to account for the correlation between \( X \) and \( Y \). However, the copula function \( c(u, v) \) is a two-dimensional non-linear function and (5) no longer applies. We make the following simplification to discretize and factorize the copula function in order to continue using the method in [17].

Suppose the PDFs of \( X \) and \( Y \) are both approximated by piecewise polynomial functions, which consist of \( I \) and \( J \) intervals, respectively. In addition, the lower and upper bounds of each interval are evenly spaced by step size \( \Delta x \) and \( \Delta y \), which satisfies \( \Delta x = \Delta y \).

\[ f_X(x) = f_X(x; i), \forall x \in [x_i, x_{i+1}), i \in \{1, \cdots, I\} \]  

(12a)

\[ f_X(x; i) = \sum_{m=1}^{M} \alpha_{im} x^{m-1} \]  

(12b)

\[ f_Y(y) = f_Y(y; j), \forall y \in [y_j, y_{j+1}), j \in \{1, \cdots, J\} \]  

(13a)

\[ f_Y(y; j) = \sum_{n=1}^{N} b_{jn} y^{n-1} \]  

(13b)

The continuous copula function \( c(\cdot) \) is discretized into \( I \cdot J \) grids over the whole domain. Note that the grid boundaries coincide with the interval boundaries of \( X \) and \( Y \), i.e., grid \(((i,j))\) is bounded by \([x_i, x_{i+1})\) and \([y_j, y_{j+1})\]. We assume \( c(\cdot) \) remains constant over each grid. Note the error due to discretization approaches 0 when \( \Delta x = \Delta y \) is adequately small. Based on (10), the discretized copula value \( c_{i,j} \) is given by the mean of the copula function over grid \(((i,j))\):
\[
\gamma_{i,j} = \int_{u_i}^{u_{i+1}} \int_{v_{j-1}}^{v_{j+1}} c(u, v) \, du \, dv \\
= C(u_{i+1}, v_j) + C(u_i, v_j) - C(u_{i+1}, v_j) - C(u_i, v_{j-1}) \\
= (u_{i+1} - u_i)(v_{j+1} - v_j)
\]

Next, the discretized copula \( \gamma_{i,j} \) is factorized into an \( x \)-axis coefficient \( \gamma_i^X \) and a \( y \)-axis coefficient \( \gamma_j^Y \), which satisfies:

\[
\gamma_{i,j} = \gamma_i^X \cdot \gamma_j^Y
\]

Therefore, the copula function is discretized and factorized into two arrays of coefficients, which apply to the per interval form of the piecewise polynomial PDFs of \( X \) and \( Y \):

\[
f_X(x; i) = \gamma_i^X \sum_{m=1}^{M} a_{im} x^{m-1}
\]

\[
f_Y(y; i) = \gamma_j^Y \sum_{n=1}^{N} b_{jn} y^{n-1}
\]

The above per interval form is then transformed into the form of Dirac delta functions and (5) can be employed to complete the convolution.

D. Evaluation metrics

As discretization and factorization are used to approximate the copula function in the proposed enhancement of convolution, the goodness-of-fit should be examined to assess whether the approximations result in significant error in the simulated results. A wide variety of tools can be used to examine the goodness-of-fit [19]. For example, graphical methods usually provide a fast and intuitive insight. In this study, we employ the probability-probability (P-P) plot to demonstrate the effects on the goodness-of-fit of the results from convolution. Given the CDFs of two distributions \( F_1(t) \) and \( F_2(t) \) defined on the same sample \( t \), the \( x \) and \( y \) coordinates of each point on the P-P plot correspond to the CDFs from two distributions \( (u_1, u_2) \), where \( u_1 = F_1(t) \) and \( u_2 = F_2(t) \). If the two distributions are identical, i.e., \( F_1(t) = F_2(t), \forall t \), then all points on the P-P plot should fall perfectly on the diagonal and any systematic deviation from the diagonal indicates differences between \( F_1(t) \) and \( F_2(t) \). In this study, the empirical distribution is used as the reference distribution and the results from convolution are compared against the reference to give a visual evaluation of the goodness-of-fit.

While graphical methods provide a fast and intuitive way, the goodness-of-fit should also be measured in a more statistically rigorous way by examining quantitative indicators. A commonly used method is the Chi-square Goodness-of-fit test [19], which groups the \( n \) collected samples into \( k \) mutually exclusive categories and denotes the observed and expected frequencies of category \( i \) by \( f_i \) and \( e_i \). The Chi-square test then examines the following criterion

\[
\chi^2 = \sum_{k=1}^{k} \frac{(f_i - e_i)^2}{e_i}
\]

which is the sum of deviations of frequencies from all categories. Under the null hypothesis \( H_0 \), which states \( F_1(t) = F_2(t) \), the total deviation \( \chi^2 \) should not exceed a critical value, which is denoted by \( \chi^2_{n, \alpha} \) and determined by the number of samples \( n \) and a pre-defined confidence level \( \alpha \). Note that although the Chi-square test statistic is used in this study, the aim is rather to use the total deviation \( \chi^2 \) as a measurement of goodness-of-fit of the convolved results than to perform the hypothesis test. When compared with the actual distributions, large \( \chi^2 \) values indicate greater deviations, hence lower goodness-of-fit.

Another evaluation metric is the coefficient of determination of the P-P plot [20], which is intended to assess the linearity of the P-P plot. Given a set of CDF pairs \( (u_1, u_2) \) from the previous P-P plot, the coefficient of determination is given by the following equation:

\[
R^2 = \frac{\text{cov}(u_1, u_2)}{\sqrt{\text{var}(u_1) \text{var}(u_2)}}^2
\]

Therefore, \( R^2 \) ranges from 0 to 1 and a higher \( R^2 \) indicates better goodness-of-fit.

III. Data

The data in this study is drawn from CAISO’s flexible capacity needs assessment studies [21], which provide one-minute time-series data for five components: total system load \( (x^t) \), power generation from wind \( (x^w) \), solar photovoltaics (PV, \( x^p \)), solar thermal \( (x^m) \), and behind-the-meter solar resources (BTM, denoted by \( x^b \)). The flexible capacity needs of CAISO are assessed yearly in pursuant to the ISO tariffs. The data used for the study, which is collected from all load serving entities (LSE), includes variable energy resources (VER), such as wind and solar. The LSE specific data is then aggregated to obtain the system-level data, which is then used by the ISO to forecast future one-minute VER power generation by scaling the collected data using the ratio of the expected future capacity over the existing capacity. Therefore, although the data is synthetic, it still preserves real-world volatility. In this study, the 2016 dataset is used.

Each year, each component includes 525,600 one-minute time-series samples, which are used to construct histograms using a bin size of 100 MW. The histograms are then approximated by piecewise linear functions and used as inputs to the method of convolution developed in Section II. In summary, the input data includes five components and their associated PDFs, which is summarized in Table I.

<table>
<thead>
<tr>
<th>Components</th>
<th>Acronyms</th>
<th>Symbols</th>
<th>PDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total system load</td>
<td>TL</td>
<td>( x^t )</td>
<td>( f^t(\cdot) )</td>
</tr>
<tr>
<td>Wind power</td>
<td>W</td>
<td>( x^w )</td>
<td>( f^w(\cdot) )</td>
</tr>
<tr>
<td>Solar PV</td>
<td>PV</td>
<td>( x^p )</td>
<td>( f^p(\cdot) )</td>
</tr>
<tr>
<td>Solar thermal</td>
<td>ST</td>
<td>( x^m )</td>
<td>( f^m(\cdot) )</td>
</tr>
<tr>
<td>Behind-the-meter</td>
<td>BTM</td>
<td>( x^b )</td>
<td>( f^b(\cdot) )</td>
</tr>
</tbody>
</table>

IV. CONVOLUTION OF TWO COMPONENTS

We start our analysis by first examining the convolution between two components only. For example, given \( f^t \) and \( f^w \), the PDFs of total load \( x^t \) and wind power \( x^w \), the method developed in Section II can be applied to approximate the PDF of their linear combination \( x^t - x^w \), which is the total load net of wind production. Additionally, the actual PDF of
$x^f - x^w$ can also be obtained directly from its histogram, which is used as a baseline in our study. Therefore, the PDFs given by convolution are compared against with the actual PDF to demonstrate the goodness-of-fit. In addition, to demonstrate the effects of correlation on convolution, the two components are convolved twice, one with the copula enhancement and the other one without. Five copula functions are employed to model the correlation across variables, including the Gaussian copula, the student’s $t$ copula, and three Archimedean copulas: Clayton, Frank and Gumbel.

Fig. 1: PDFs of the linear combination of two components: the results from convolution (with and without correlation accounted for) and the actual PDF. Note that although five copula functions are studied, only the Gaussian copula is presented for better visibility.

The results are demonstrated in Figs. 1 and 2. The PDFs given by convolution (both with and without correlation accounted for) are compared against the baseline, as shown in Fig. 1. While the PDFs given by convolution agree well with the baseline most of the time, significant disparities are observed in Fig. 1b. This is also reflected in the P-P plot in Fig. 2b, where the profile of convolution presents greater deviation from linearity. Therefore, both figures suggest greater incompatibility between the results from convolution and the actual distribution when zero correlation is assumed. This is further supported by the evaluation metrics. As shown in Tables II and III, the convolution between $x^f$ and $x^p$ presents the lowest $R^2$ value and the highest $\chi^2$ value when their correlation is not accounted for.

In addition, both visual inspection and the evaluation metrics indicate that the copula enhancement can effectively improve the convolution by reducing deviation from actual distributions. As illustrated in Figs. 1b and 2b, the results from the copula enhanced convolution exhibit better agreement with the baseline. While the results in both Figs. 1 and 2 only include the Gaussian copula, Tables II and III give a comprehensive comparison by including other types of copula functions. It shows that the copula enhanced convolution always presents higher $R^2$ and lower $\chi^2$ than that when correlation is not included. However, it also suggests the optimal copula function may vary. For example, the student’s $t$ copula results in the optimal results for $x^f - x^p$ and $x^f - x^t$, while the Archimedean copulas are preferred when $x^w$ or $x^m$ is convolved with $x^f$.

We note here that in the $\chi^2$ test, the number of samples $n = 525,600$, hence the degree of freedom of the $\chi^2$ distribution is $n - 1 = 525,599$. Therefore, the critical value beyond which the null hypothesis $H_0$ will be rejected is $\chi^2_{n-1, \alpha} = 527,287$, given the confidence level $1 - \alpha = 0.95$ and Table III indicates that all results pass the $\chi^2$ test.

TABLE II: The coefficients of determination ($R^2$). The shaded cells indicate the maximum $R^2$ value in that column.

<table>
<thead>
<tr>
<th>Copula</th>
<th>$x^f - x^w$</th>
<th>$x^f - x^p$</th>
<th>$x^t - x^t$</th>
<th>$x^f - x^m$</th>
<th>$x^w - x^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No copula</td>
<td>0.99978</td>
<td>0.99968</td>
<td>0.99990</td>
<td>0.99992</td>
<td>0.99753</td>
</tr>
<tr>
<td>Gaussian</td>
<td>0.99979</td>
<td>0.99903</td>
<td>0.99998</td>
<td>0.99905</td>
<td>0.99762</td>
</tr>
<tr>
<td>$t$</td>
<td>0.99984</td>
<td>0.99968</td>
<td>0.99992</td>
<td>0.99992</td>
<td>0.99751</td>
</tr>
<tr>
<td>Clayton</td>
<td>0.99980</td>
<td>0.99894</td>
<td>0.99997</td>
<td>0.99997</td>
<td>0.99753</td>
</tr>
<tr>
<td>Frank</td>
<td>0.99978</td>
<td>0.99968</td>
<td>0.99992</td>
<td>0.99992</td>
<td>0.99751</td>
</tr>
<tr>
<td>Gumbel</td>
<td>0.99984</td>
<td>0.99968</td>
<td>0.99992</td>
<td>0.99992</td>
<td>0.99751</td>
</tr>
</tbody>
</table>

TABLE III: The $\chi^2$ test statistics. The shaded cells indicate the minimum $\chi^2$ value in that column.

<table>
<thead>
<tr>
<th>Copula</th>
<th>$x^f - x^w$</th>
<th>$x^f - x^p$</th>
<th>$x^t - x^t$</th>
<th>$x^f - x^m$</th>
<th>$x^w - x^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No copula</td>
<td>14124.3</td>
<td>49972.4</td>
<td>6531.2</td>
<td>5855.9</td>
<td>40517.3</td>
</tr>
<tr>
<td>Gaussian</td>
<td>13650.1</td>
<td>17275.6</td>
<td>3599.0</td>
<td>4250.1</td>
<td>13014.2</td>
</tr>
<tr>
<td>$t$</td>
<td>13645.6</td>
<td>15227.9</td>
<td>3595.0</td>
<td>4207.6</td>
<td>-</td>
</tr>
<tr>
<td>Clayton</td>
<td>10165.2</td>
<td>49972.5</td>
<td>6531.2</td>
<td>5855.9</td>
<td>43774.9</td>
</tr>
<tr>
<td>Frank</td>
<td>13611.6</td>
<td>17071.0</td>
<td>3649.1</td>
<td>3945.3</td>
<td>12141.8</td>
</tr>
<tr>
<td>Gumbel</td>
<td>14124.4</td>
<td>49972.6</td>
<td>6531.2</td>
<td>5855.9</td>
<td>40517.6</td>
</tr>
</tbody>
</table>

V. CONVOLUTION OF MULTIPLE COMPONENTS

Using the notations from Table I, net load $x^n$ is given by subtracting all VER power generation from the total load: $x^n = x^f - x^w - x^p - x^t - x^m$. While the method developed in Section II is designed to convolve at most two components at a time, the VER components are convolved with total load iteratively, i.e., each time only one VER component is
convolved with the total load $x^f$ and the procedure is repeated four times. Note that in the copula enhanced convolution, the copula function is recalculated during each iteration to reflect the updated $x^f$ term. This method can be easily extended to as many components. Similar to the two-component case, the actual distributions of net load are used to benchmark the results from convolution, both with and without the copula enhancement. Note that from the results in the previous section, the optimal copula function may vary depend on the convolved components. For simplicity, when the VER components are convolved in order, the correlations are modeled using one type of copula function only. All five copulas in Section IV are tested.

The results are shown in Fig. 3, Tables II and III. Fig. 3 provides an intuitive view of the comparison between convolution and the actual distribution. Similar to the two-component case, convolution with copula enhancement exhibits better agreement with the actual distribution as presented in Fig. 3a and stronger linearity in Fig. 3b, indicating improvement due to the inclusion of correlation. This is also supported by the evaluation metrics listed in Tables II and III, where the $\chi^2$ statistic from convolution with copula enhancement presents over 70% reduction compared with the results when the correlation is not accounted for.

![Image](a) Probability density functions.

![Image](b) P-P plot.

Fig. 3: Graphic results from the multiple component convolution. (a) Profiles of PDFs. (b) P-P plots. Note that although five copula functions are studied, only the Gaussian copula is presented for better visibility.

VI. CONCLUSION

This study presents a numerical technique to estimate the distribution of the linear combination of two correlated random variables based on the convolution of two piecewise polynomial functions. The correlations are accounted for by using copulas. With the piecewise polynomial representation and copula enhancement, this technique can be applied to any two correlated random variables of any types of distribution functions. In addition, we also demonstrate how to extend the method to estimate the distribution of multiple correlated random variables.

Using the dataset from CAISO’s flexible capacity needs assessment, the developed method is applied to estimate the distribution of two and multiple components. By comparing with the actual distributions, our results demonstrate adequate accuracy to be used in real-world applications, which may include estimating the distributions of net load given probabilistic forecasts of load and VER outputs.

ACKNOWLEDGMENT

This work is supported by the U.S. Department of Energy under the Prime Contract No. DEEE0008215. The authors would thank Clyde Loutan, Rebecca Webb and Amber Motley from CAISO for their insights. Any views, thoughts, and opinions expressed in the text belong solely to the authors.

REFERENCES