

Logical and Physical Reversibility of Conservative Skyrmion Logic

Xuan Hu^{1*}, Benjamin W. Walker¹, Felipe García-Sánchez^{2*}, Alexander J. Edwards^{1**}, Peng Zhou^{1**}, Jean Anne C. Incorvia^{3***}, Alexandru Paler⁴, Michael P. Frank^{5*}, and Joseph S. Friedman^{1**}

¹University of Texas at Dallas, Richardson, TX 75080, USA

²Universidad de Salamanca, 37008 Salamanca, Spain

³University of Texas at Austin, Austin, TX 78712, USA

⁴Aalto University, 02150 Espoo, Finland

⁵Sandia National Laboratories, Albuquerque, NM 87185, USA

* Member, IEEE

** Graduate Student Member, IEEE

*** Senior Member, IEEE

Received 31 Mar 2022, revised 25 Apr 2022, accepted 30 Apr 2022, published 11 May 2022, current version 20 Jun 2022.

Abstract—Magnetic skyrmions are nanoscale whirls of magnetism that can be propagated with electrical currents. The repulsion between skyrmions inspires their use for reversible computing based on the elastic billiard ball collisions proposed for conservative logic in 1982. In this letter, we evaluate the logical and physical reversibility of this skyrmion logic paradigm, as well as the limitations that must be addressed before dissipation-free computation can be realized.

Index Terms—Spin electronics, reversible computing, conservative logic, spintronics, skyrmion, voltage-controlled magnetic anisotropy.

I. INTRODUCTION

The conventional *nonreversible* paradigm for general digital computing will eventually approach fundamental thermodynamic limits on its energy efficiency, which stem ultimately from the fact that this standard paradigm relies primarily on operations that systematically discard correlated logical information, and therefore increase entropy [Frank 2005, 2018, 2021]. For example, a typical digital logic architecture in complementary metal-oxide semiconductor (CMOS) destructively overwrites the output of each active logic gate with a new bit value on each clock cycle. It has been known ever since the field of the thermodynamics of computation was first elucidated by Landauer [1961] and Bennett [1973, 1982, 2003] that general digital computing technologies can only avoid the consequent limits on their energy efficiency if they are instead re-architected on the basis of *logically reversible* operations that are implemented (at the device and circuit level) in a nearly *thermodynamically reversible* way [Frank 2017].

Logical reversibility requires logical operations to conserve information content, and is a prerequisite for thermodynamic reversibility. In a thermodynamically or physically reversible system, the entropy of the environment does not increase during computation, enabling incredible energy savings. While complete physical reversibility is not attainable in any nonequilibrium system at nonzero temperature, systems can be designed to approach these thermodynamic bounds.

Although this alternative paradigm of *reversible computing* can be implemented in CMOS technology [Younis 1993; Frank 2020a, 2020b], there is also a need to explore a range of other, novel device

Corresponding author: Alexander J. Edwards (e-mail: alexander.edwards@utdallas.edu).

This letter has supplementary downloadable material available at <https://doi.org/10.1109/LMAG.2022.3174514>, provided by the authors. Digital Object Identifier 10.1109/LMAG.2022.3174514

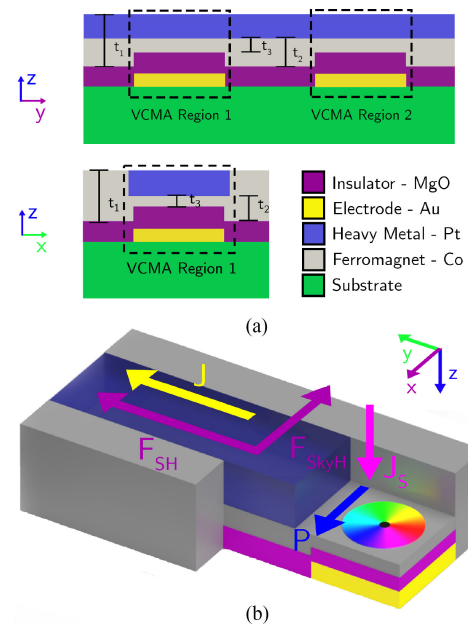


Fig. 1. Skyrmion logic track structure. (a) Synchronizer cross sections in the yz and xz planes. The combined track thickness is 1.6 nm, with a track width of 20 nm. (b) A Neel skyrmion (colored circle) exists in the ferromagnetic layer. Injected electric current in the $+y$ direction (J) induces a spin current (J_s) with polarization P in the $+z$ direction via the spin-Hall effect. This spin current produces a $+y$ -directed spin-Hall force (F_{SH}), propelling the skyrmion in the $+y$ direction. The skyrmion-Hall (F_{SkyH}) effect creates an x -directed force, which deviates the skyrmion from its current trajectory unless a track is present to provide repulsion.

technologies to determine whether they may potentially exhibit advantages for reversible computing compared to CMOS [Frank 1999]. In the line of work reported here, magnetic skyrmions are being explored

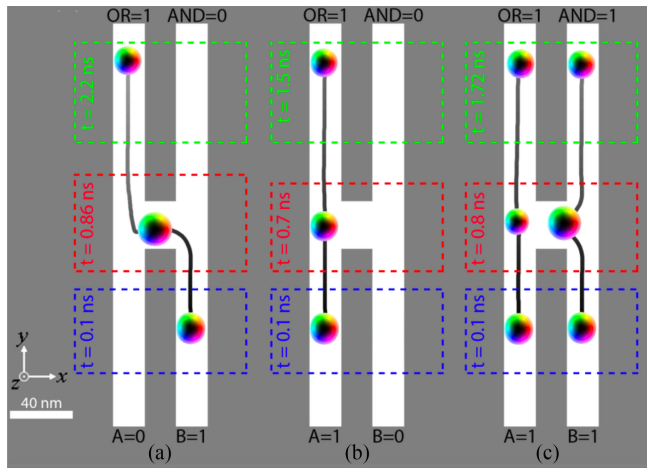


Fig. 2. Micromagnetic simulation results for AND/OR gate with current (5×10^{10} A/m²) in the $+y$ direction, with skyrmion trajectory represented by the black path. (a) $A = 0$, $B = 1$. (b) $A = 1$, $B = 0$. (c) $A = 1$, $B = 1$. All simulations are performed with MuMax3 [Vansteenkiste 2014]. The SOT generated by the uniform electrical current was simulated as an external field to induce skyrmion motion. The Oersted field was excluded from the simulations, as its effect is insignificant relative to that of the Dzyaloshinskii–Moriya exchange interaction. Simulations were run at 0 K. Similar logic and system functionality is expected at nonzero temperatures, as both skyrmion transport and VCMA-based pinning are robust to thermal noise [Yu 2017, Walker 2021], though correct operation may require adjustments to material composition, current distribution, and geometry.

as a candidate device technology for the implementation of reversible computing.

Magnetic skyrmions have garnered recent interest in the physics and computing communities due to their nonvolatility, small size, and ability to be moved with an electrical current [Jiang 2017a, Everschor-Sitte 2018]. They have been proposed for a variety of computational schemes, including reservoir computing [Pinna 2020], stochastic computing [Pinna 2018], and Brownian token-based computing [Nozaki 2019], all of which are based on stochastic dynamics and are not necessarily well-suited for use in general-purpose high-performance deterministic digital computing. In contrast, our logic paradigm leverages the nonvolatility of the skyrmions while minimizing thermal stochasticity to approach Landauer’s thermodynamic limit for deterministic logical computation.

II. LOGICALLY REVERSIBLE SKYRMION LOGIC

Skyrmions are topologically stable magnetic quasiparticles that can be propelled with applied electrical current or pinned with voltage-controlled magnetic anisotropy (VCMA) [Wang 2018]. Their mobility paired with the skyrmion-Hall effect [Jiang 2017b] and skyrmion–skyrmion repulsion allows for the construction of a skyrmion-based logic system [Chauwin 2019] based on billiard ball computing [Fredkin 1982]. Fig. 1 shows the structure of such a system, where a skyrmion exists inside the Cobalt layer due to the Dzyaloshinskii–Moriya interaction between the ferromagnet and heavy metal. Encoding a logical “1” or “0” as the presence or absence of a skyrmion, respectively, the skyrmion logic scheme manipulates these forces to create logic functions such as the AND/OR gate, the Ressler–Feynman

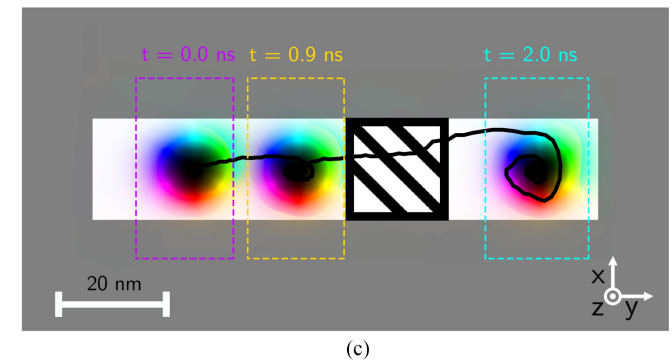
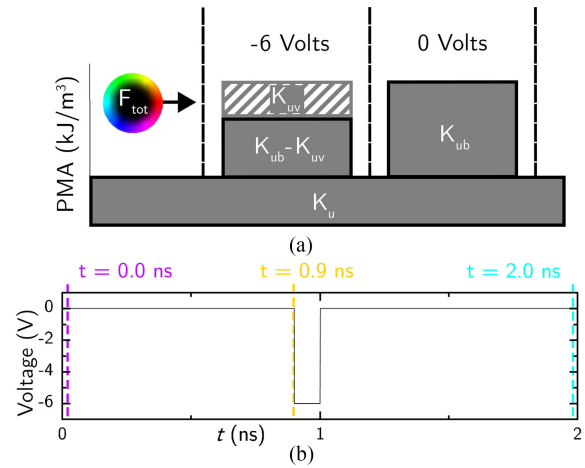


Fig. 3. VCMA synchronizer with a constant injected current of 5×10^{10} A/m² in the $+y$ direction. (a) Illustration of the skyrmion synchronization method. The skyrmion is depinned when $F_{\text{tot}} > K_{ub} + K_{uv}$. (b) Voltage applied to the VCMA region over one clock cycle. (c) Micromagnetic simulation results. Skyrmion trajectory is represented by the black line, and the VCMA region is represented by the dashed square. The positive anisotropy barrier pins the skyrmion at $t = 0.9$ ns, and the negative voltage at $t = 1.0$ ns reduces the barrier, depinning the skyrmion. At nonzero temperatures, the electric field must be larger than at 0 K to ensure pinning in the presence of noise [Walker 2021].

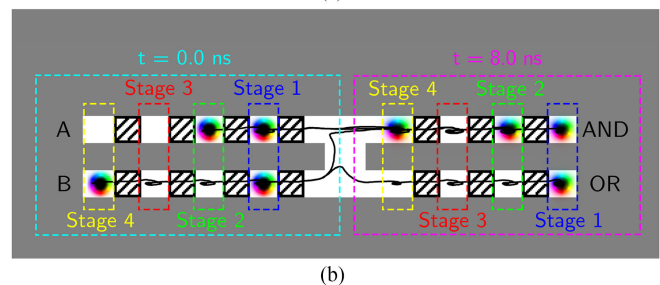
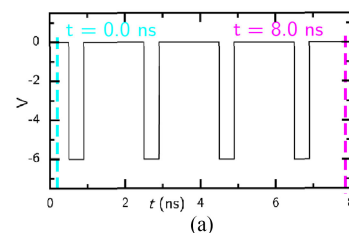


Fig. 4. VCMA synchronization of pipelined skyrmion AND/OR gate, with a constant current (5×10^{10} A/m²) injected in the $+y$ direction. (a) Clock waveform applied to the VCMA synchronizers. (b) Micromagnetic simulation results for pipelined AND/OR gate, demonstrating proper logical operation.

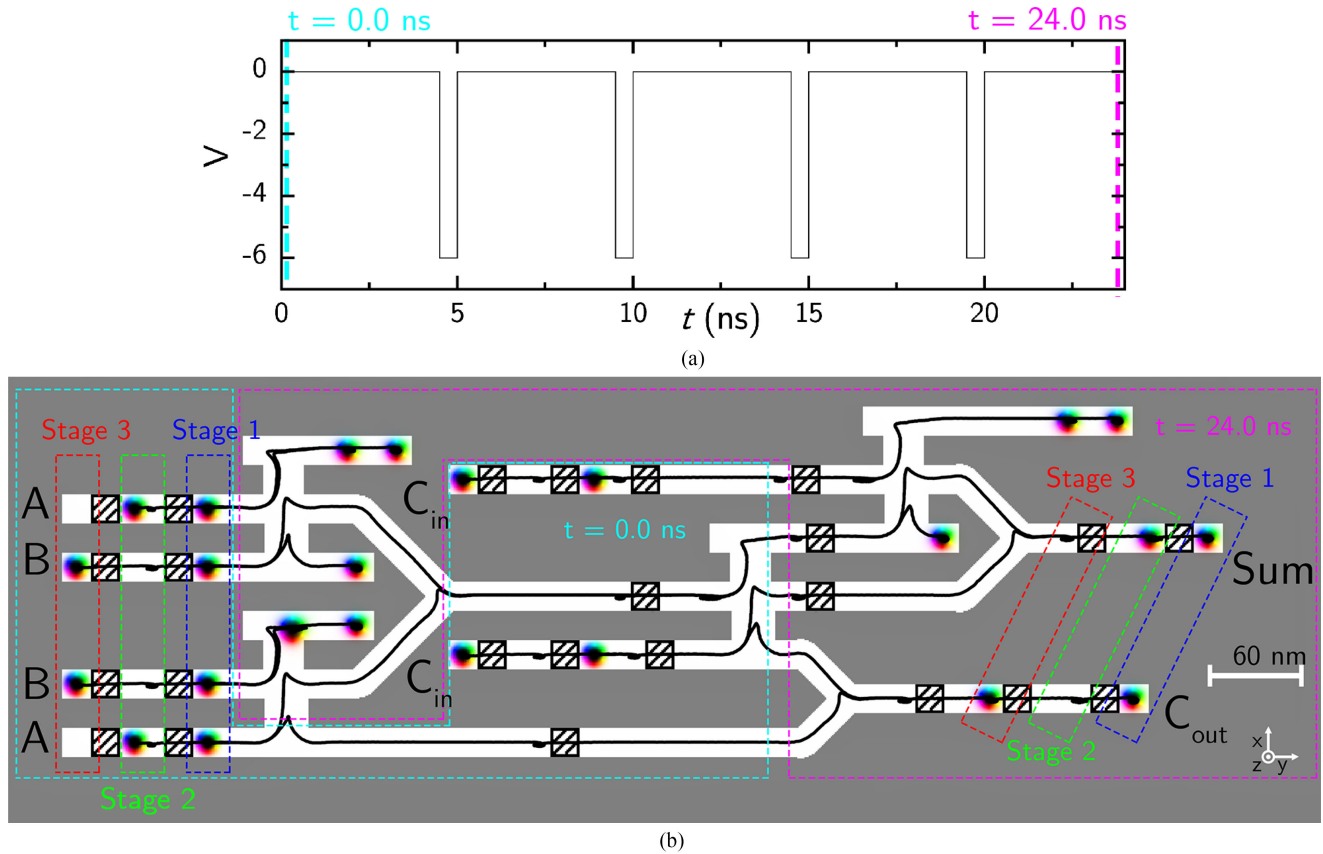


Fig. 5. VCMA synchronization of pipelined skyrmion one-bit full-adder with a constant current (5×10^{10} A/m²) injected in the $+y$ direction. (a) Clock waveform applied to the VCMA synchronizers. (b) Micromagnetic simulation results for pipelined one-bit full-adder, demonstrating proper logical operation. Note that in a real system, the dead-ends would be replaced with out-of-plane tracks for cascaded logic.

switch gate [Ressler 1981], and one-bit full adders. This proposed logic scheme conserves skyrmions, and its structures can exhibit logical reversibility.

Four forces underlie this conservative skyrmion computing scheme (Fig. 1). Uniform electrical current through the heavy metal layer in the $+y$ direction moves the skyrmion in the same direction via the spin-Hall effect. The skyrmion's velocity causes it to deviate in a direction perpendicular to its motion ($-x$) through the skyrmion-Hall effect [Jiang 2017b]. Track-edge repulsion allows the skyrmion to move in a straight line despite the skyrmion-Hall effect, and skyrmion-skyrmion repulsion provides the billiard ball-like interaction necessary for logic [Jiang 2017a, Everschor-Sitte 2018, Chauwin 2019, Walker 2021].

The skyrmion AND/OR gate uses these forces for conditionally reversible computation (Fig. 2). Electronic current moves the skyrmions in a straight line in the $+y$ direction due to track repulsion. However, when a skyrmion in the right track reaches the gate junction, the loss of track repulsion allows it to move in the $-x$ direction into the left lane [Fig. 2(a)]. Skyrmions in the left lane remain in the left lane, due to constant track repulsion [Fig. 2(b)]. Importantly, when two skyrmions are synchronized in both tracks [Fig. 2(c)], the skyrmion-skyrmion repulsion allows for the right skyrmion to remain in the right track. The output of the left track therefore represents logical OR, while the right represents logical AND. Due to the input ambiguity of the (OR = 1, AND = 0) case, the AND/OR gate is conditionally logically reversible [Frank 2017] if one of the two input cases, ($A = 0, B = 1$) or

($A = 1, B = 0$), is excluded from operation, allowing a version of this gate to have an implementation that approaches the thermodynamic limit. However, a reversible version of this particular gate would not be useful; later, we discuss a more useful reversible gate.

As skyrmions need to be synchronized when entering a gate for proper logical operation, varying skyrmion path lengths and thermal fluctuations can result in logical errors. Through VCMA clocking [Walker 2021], the application of an electric field ($+z$) via an electrode changes the perpendicular magnetic anisotropy (PMA) of the track. As skyrmions can be pinned by a PMA gradient, the voltage modulation of the electrode at the VCMA region (Fig. 3) can be used to pin and depin skyrmions as a clocking mechanism [Wang 2018]. These synchronizers are used to create a pipelined AND/OR gate in Fig. 4.

When multiple skyrmion gates are cascaded together, complex functions can be realized, such as the full-adder. In Fig. 5, an XOR gate of A and B is created by connecting two Ressler-Feynman switch gates in parallel. Using VCMA-based synchronization, this XOR output is then synchronized with C_{in} before passing through two additional switch gates and a synchronizer to result in Sum and C_{out} .

III. PHYSICAL REVERSIBILITY OF SKYRMION LOGIC

The inclusion of convergent skyrmion paths to perform the OR operation in both the skyrmion AND/OR gate and full-adder preclude

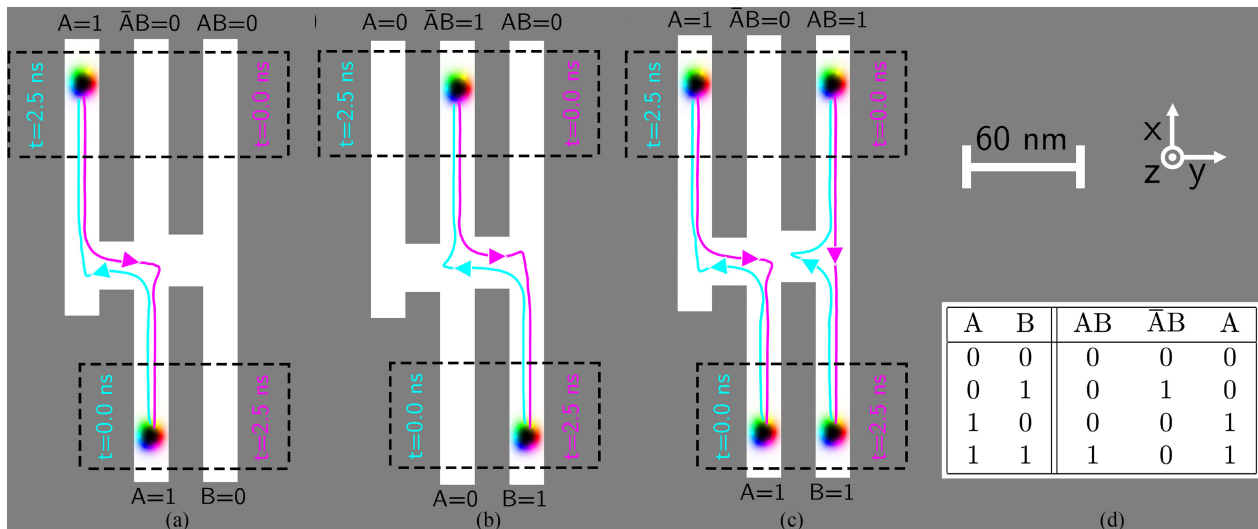


Fig. 6. Ressler-Feynman switch gate with current ($\pm 5 \times 10^{10}$ A/m²). Logic shown for input combinations. (a) A = 1, B = 0. (b) A = 0, B = 1. (c) A = 1, B = 1. Skyrmion trajectory under positive and negative current is represented by cyan and magenta colored paths, respectively. The forward and reverse current trajectories vary slightly, demonstrating hysteresis. (d) Ressler-Feynman switch gate truth table.

these gates from being physically reversible. One indication of this is that if a gate is run in reverse, its outputs do not always return to their input positions; this implies information—and therefore energy—loss. The next step toward total physical reversibility is to implement logic gates, which may be reversed, with information carriers returning from primary *outputs* to primary *inputs*.

As seen in Fig. 6 and supplementary video 1, the skyrmion Ressler-Feynman switch gate achieves this step toward reversibility: when there is one skyrmion in either input track [Fig. 6(c) and (d)], the skyrmion deviates in the $+x$ direction into the adjacent leftward track due to the skyrmion-Hall effect. When both input skyrmions are present [Fig. 6(e)], the skyrmions repel each other, forcing the B skyrmion to remain in the right track. Likewise, under reverse current conditions, the $-x$ deviation causes the skyrmions to return to their respective inputs, demonstrating reversibility in this (limited) sense.

This is not full physical reversibility, however, as the skyrmion trajectories differ between the forward and reverse directions, indicating hysteresis—which inherently incurs an energy cost. The skyrmion path length variation necessitates synchronization, destroying information and dissipating energy. Additionally, the magnetic damping that underlies controllable skyrmion motion further contributes to the energetic dissipation within the magnetic lattice. While these effects can be minimized through the selection of improved materials and device design, they inhibit the full physical reversibility of this system. Despite these energetic losses, reversible skyrmion logic remains extremely energy efficient [Walker 2021].

As the Ressler-Feynman switch gate is functionally complete, switch gates can be cascaded to implement any arbitrary Boolean function, including a physically reversible full-adder. Therefore, large-scale skyrmion logic circuits that can be reversed may be constructed from these Ressler-Feynman switch gates, enabling a path toward total physical reversibility.

IV. CONCLUSION

Although the scheme can exhibit logical reversibility, more work is needed to achieve total physical reversibility. In particular, at present: 1) the forward and reverse trajectories differ slightly, exhibiting hysteresis, which is a hallmark of dissipative processes; 2) the explicit synchronization operations discard physical information in the form of accumulated timing uncertainty; and 3) the driving currents will necessarily be dissipative unless superconducting materials can be used. However, research on this technology is still in its infancy, and it holds potential for further improvement.

ACKNOWLEDGMENT

The authors thank E. Laws, J. McConnell, N. Nazir, L. Philoon, and C. Simmons for technical support, and the Texas Advanced Computing Center at The University of Texas at Austin, Austin, TX, USA, for providing computational resources.

This work was supported in part by the Advanced Simulation and Computing Program at the U.S. Department of Energy's National Nuclear Security Administration (NNSA), in part by National Science Foundation under CCF Award 1910800, and in part by the Texas Analog Center of Excellence Undergraduate Internship Program. Sandia National Laboratories is a multimission laboratory managed and operated by National Technology and Engineering Solutions of Sandia, LLC, a wholly owned subsidiary of Honeywell International, Inc., for NNSA under contract DE-NA0003525. This document describes objective technical results and analysis. Any subjective views or opinions that might be expressed in this document do not necessarily represent the views of the U.S. Department of Energy or the United States Government. Approved for public release SAND2022-5262 J.

This research is supported in part by the National Science Foundation under CCF award 1910800 and the Texas Analog Center of Excellence undergraduate internship program.

REFERENCES

- Bennett C H (1973), "Logical reversibility of computation," *IBM J. Res. Develop.*, vol. 17, pp. 525–532, doi: [10.1147/rd.176.0525](https://doi.org/10.1147/rd.176.0525).
- Bennett C H (1982), "The thermodynamics of computation—A review," *Int. J. Theor. Phys.*, vol. 21, pp. 905–940, doi: [10.1007/BF02084158](https://doi.org/10.1007/BF02084158).
- Bennett C H (2003), "Notes on Landauer's principle, reversible computation, and Maxwell's demon," *Stud. Hist. Philos. Sci. Part B: Stud. Hist. Philos. Modern Phys.*, vol. 34, pp. 501–510, doi: [10.1016/S1355-2198\(03\)00039-X](https://doi.org/10.1016/S1355-2198(03)00039-X).

- Chauwin M, Hu X, Garcia-Sanchez F, Betrabet N, Paler A, Moutafis C, Friedman J S (2019), "Skyrmion logic system for large-scale reversible computation," *Phys. Rev. Appl.*, vol. 12, 064053, doi: [10.1103/PhysRevApplied.12.064053](https://doi.org/10.1103/PhysRevApplied.12.064053).
- Everschor-Sitte K, Masell J, Reeve R M, Kläui M (2018), "Perspective: Magnetic skyrmions—Overview of recent progress in an active research field," *J. Appl. Phys.*, vol. 124, 240901, doi: [10.1063/1.5048972](https://doi.org/10.1063/1.5048972).
- Frank M P (1999), "Reversibility for efficient computing," Ph.D. dissertation, Dept. Elect. Eng. Comput. Sci., Massachusetts Inst. Technol., Cambridge, MA, USA. [Online]. Available: <https://hdl.handle.net/1721.1/9464>
- Frank M P (2005), "Introduction to reversible computing: Motivation, progress, and challenges," in *Proc. 2nd Conf. Comput. Front.*, pp. 385–390, doi: [10.1145/1062261.1062324](https://doi.org/10.1145/1062261.1062324).
- Frank M P (2017), "Foundations of generalized reversible computing," in *Proc. Int. Conf. Reversible Comput.*, pp. 19–34, doi: [10.1007/978-3-319-59936-6_2](https://doi.org/10.1007/978-3-319-59936-6_2).
- Frank M P (2018), "Physical foundations of Landauer's principle," in *Reversible Computation*, Kari J and Ulidowski I, Eds, Cham, Switzerland: Springer, pp. 3–33, doi: [10.1007/978-3-319-99498-7_1](https://doi.org/10.1007/978-3-319-99498-7_1).
- Frank M P, Brocato R W, Conte T M, Hsia A H, Jain A, Missert N A, Shukla K, Tierney B D (2020a), "Special session: Exploring the ultimate limits of adiabatic circuits," in *Proc. IEEE 38th Int. Conf. Comput. Des.*, pp. 21–24, doi: [10.1109/ICCD50377.2020.00018](https://doi.org/10.1109/ICCD50377.2020.00018).
- Frank M P, Brocato R W, Tierney B D, Missert N A, Hsia A H (2020b), "Reversible computing with fast, fully static, fully adiabatic CMOS," in *Proc. Int. Conf. Rebooting Comput.*, pp. 1–8, doi: [10.1109/ICRC2020.2020.00014](https://doi.org/10.1109/ICRC2020.2020.00014).
- Frank M P, Shukla K (2021), "Quantum foundations of classical reversible computing," *Entropy*, vol. 23, 701, doi: [10.3390/e23060701](https://doi.org/10.3390/e23060701).
- Fredkin E, Toffoli T (1982), "Conservative logic," *Int. J. Theor. Phys.*, vol. 21, pp. 219–253, doi: [10.1007/BF01857727](https://doi.org/10.1007/BF01857727).
- Jiang W, Chen G, Liu K, Zang J, te Velthuis S G, Hoffmann A (2017a), "Skyrmions in magnetic multilayers," *Phys. Rep.*, vol. 704, pp. 1–49, doi: [10.1016/j.physrep.2017.08.001](https://doi.org/10.1016/j.physrep.2017.08.001).
- Jiang W, Zhang X, Yu G, Zhang W, Wang X, Jungfleisch M B, Pearson J E, Cheng X, Heinonen O, Wang K L, Zhou Y, Hoffmann A, te Velthuis S G E (2017b), "Direct observation of the skyrmion Hall effect," *Nature Phys.*, vol. 13, pp. 162–169, doi: [10.1038/nphys3883](https://doi.org/10.1038/nphys3883).
- Landauer R (1961), "Irreversibility and heat generation in the computing process," *IBM J. Res. Develop.*, vol. 5, pp. 183–191, doi: [10.1147/rd.53.0183](https://doi.org/10.1147/rd.53.0183).
- Nozaki T, Jibiki Y, Goto M, Tamura E, Nozaki T, Kubota H, Fukushima A, Yuasa S, Suzuki Y (2019), "Brownian motion of skyrmion bubbles and its control by voltage applications," *Appl. Phys. Lett.*, vol. 114, 012402, doi: [10.1063/1.5070101](https://doi.org/10.1063/1.5070101).
- Pinna D, F Abreu Araujo, Kim J-V, Cros V, Querlioz D, Bessiere P, Droulez J, Grollier J (2018), "Skyrmion gas manipulation for probabilistic computing," *Phys. Rev. Appl.*, vol. 9, 064018, doi: [10.1103/PhysRevApplied.9.064018](https://doi.org/10.1103/PhysRevApplied.9.064018).
- Pinna D, Bourianoff G, Everschor-Sitte K (2020), "Reservoir computing with random skyrmion textures," *Phys. Rev. Appl.*, vol. 14, 054020, doi: [10.1103/PhysRevApplied.14.054020](https://doi.org/10.1103/PhysRevApplied.14.054020).
- Ressler A L (1981), "The design of a conservative logic computer and a graphical editor simulator," Ph.D. dissertation, Elect. Eng. Comput. Sci., Massachusetts Inst. Technol. Cambridge, MA, USA. [Online]. Available: <https://hdl.handle.net/1721.1/15895>
- Vansteenkiste A, Leliaert J, Dvornik M, Helsen M, Garcia-Sanchez F, B. Van Waeyenberge (2014), "The design and verification of MuMax3," *AIP Adv.*, vol. 4, 107133, doi: [10.1063/1.4899186](https://doi.org/10.1063/1.4899186).
- Walker B W, Cui C, Garcia-Sanchez F, Incorvia J A C, Hu X, Friedman J S (2021), "Skyrmion logic clocked via voltage-controlled magnetic anisotropy," *Appl. Phys. Lett.*, vol. 118, 192404, doi: [10.1063/5.0049024](https://doi.org/10.1063/5.0049024).
- Wang J, Xia J, Zhang X, Zhao G P, Ye L, Wu J, Xu Y, Zhao W, Zou Z, Zhou Y (2018), "Controllable transport of a skyrmion in a ferromagnetic narrow channel with voltage-controlled magnetic anisotropy," *J. Phys. D: Appl. Phys.*, vol. 51, 205002, doi: [10.1088/1361-6463/aab927](https://doi.org/10.1088/1361-6463/aab927).
- Younis S G, Knight Jr T F (1993), "Practical implementation of charge recovering asymptotically zero power CMOS," in *Proc. Symp. Res. Integr. Syst.*, pp. 234–250, doi: [10.5555/163429.163468](https://doi.org/10.5555/163429.163468).
- Yu G, Upadhyaya P, Shao Q, Wu H, Yin G, Li X, He C, Jiang W, Han X, Amiri P K, Wang K L (2017), "Room-temperature skyrmion shift device for memory application," *Nano Lett.*, vol. 17, pp. 261–268, doi: [10.1021/acs.nanolett.6b04010](https://doi.org/10.1021/acs.nanolett.6b04010).