Calculation of the bit-error ratio in wavelength-division-multiplexed return-to-zero systems when the nonlinear penalty is dominated by collision-induced timing jitter

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Abstract: We analyze the accuracy of a simple approach for calculating the BER in WDM-RZ systems, where the principal nonlinear impairment is collision-induced timing jitter. This approach produces BERs that are accurate to within one order of magnitude.

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1. Introduction

Analytical evaluation of the bit error ratio (BER) in optical fiber communications systems in the presence of the strong patterning effects due to nonlinearity has attracted considerable attention [1]–[5], particularly for the non-return-to-zero (NRZ) transmission format [1]–[3], [5]. The basic idea in these approaches is to utilize the pump-probe method and calculate the cross-phase-modulation-induced distortion using a small signal analysis. In order to determine the influence of the nonlinearity on the system performance, the cross-phase-modulation-induced distortion is treated as additive Gaussian noise and a correction to the Q-factor is calculated.

Until recently [6], there has been no description of accurate methods for calculating the BER in wavelength-division-multiplexed (WDM) return-to-zero (RZ) systems with strong nonlinear patterning effects that has appeared in the literature. In WDM-RZ systems, the major manifestation of fiber nonlinearity is collision-induced timing jitter due to interchannel cross-phase modulation [7], [8]. In order to account for this timing jitter when calculating the BER, it is necessary to know the probability density function (pdf) of the time shift of a pulse. Previously, we have computed this pdf using the characteristic function method and showed that it can be used in an accurate calculation of the pdf of the received current, and hence of the BER [6]. It is natural to approximate the pdf of the time shift with a Gaussian function since the central limit theorem implies that the pdf approaches a Gaussian function in the limit of a large number of pulse collisions. The Gaussian approximation significantly simplifies the analysis and reduces the computational effort.

In this work, we calculate the pdf of the time shift using a Gaussian approximation and show that it is a poor fit to the true pdf in the low-probability tails of the pdf, but it may nevertheless yield a reasonably good estimate of the BER when both collision-induced timing jitter and amplified spontaneous emission (ASE) noise are present. However, for small enough channel spacings to lead to strong nonlinear crosstalk and high BERs in the absence of forward error correction, the exact pdf of the time shift still must be used for the calculation of the BER.

2. Gaussian model for the pdf of collision-induced time shift

In order to obtain the time shift pdf, we begin by calculating the time shift function [9]. This function describes the time shift of an optical pulse in one WDM channel due to its interaction with a pulse in another channel, as a function of the initial time separation of the pulses and the wavelength separation of the channels. We assume that the total time shift of a pulse is the sum of the time shifts caused by individual pairwise collisions. Although not obvious for modern RZ systems, in which a pulse overlaps with several of its neighbors due to a large dispersive spread, this assumption has proved to be valid in both soliton and quasi-linear systems [6], [7]. We refer to the pulse with respect to which we are calculating the time shift function as the target pulse and the channel in which it is located as the probe channel. The other WDM channels are referred to as the pump channels. We define the time shift function as $\tau(\Delta f_k, l) \equiv \tau_{kl}$, where l is the initial offset of a bit slot in a pump channel that is separated by Δf_k from the probe channel k. We compute this time shift using a semi-analytical method that takes into account gain and loss, as well as the

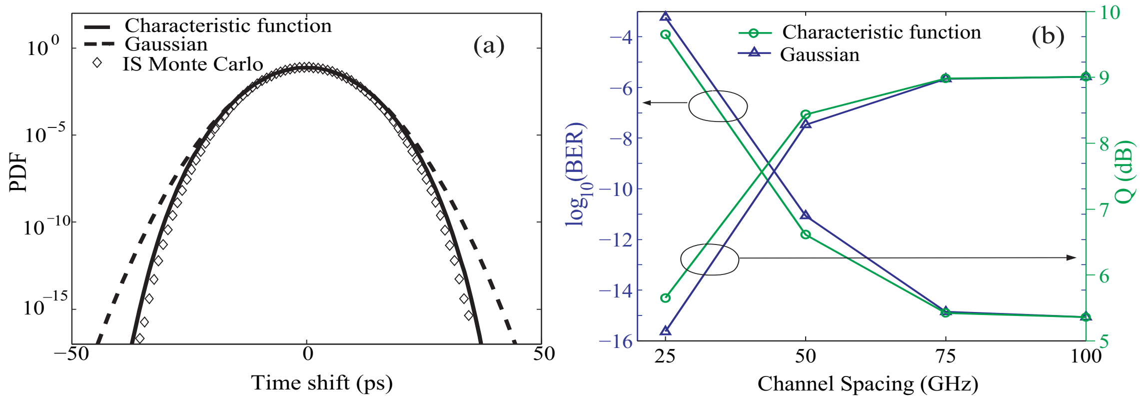

Fig. 1. (a) Pdf of collision-induced time shift for the 50 GHz channel spacing and (b) BER and Q vs. channel spacing

nonlinear interactions and dispersion [7], [9]. For a particular pattern of 1's and 0's, the total time shift of the target pulse is

$$T_{\text{total}} = \sum_{kl} \alpha_{kl} \tau_{kl}, \tag{1}$$

where $\alpha_{kl} = 1$ if the l-th bit slot in the k-th channel contains a pulse, corresponding to a digital 1, and is zero otherwise.

We assume that the transmitted data is random so that the α_{kl} are independent, identically distributed random variables, each having probability 1/2 of being 1 or 0. The number of terms in the sum (1) is finite due to the finite number of pulses that the target pulse collides with during propagation. On the other hand, since the number of collisions is large, it is reasonable to assume that T_{total} is Gaussian-distributed by the central limit theorem. In this case, it is sufficient to know the mean μ_T and variance σ_T^2 of T_{total}. In the absence of higher-order dispersion [9],

$$\mu_T = \sum_{kl} \tau_{kl} = 0 \qquad \text{and} \qquad \sigma_T^2 = \langle T_{\text{total}}^2 \rangle = \frac{1}{4} \sum_{kl} \tau_{kl}^2. \tag{2}$$

We calculated the pdfs of the collision-induced time shift for a prototypical long-haul undersea system. We consider a 10 Gb/s system with a propagation distance of approximately 5000 km [10]. The transmission line included 100 periods of the dispersion map consisting of 34 km of D_+ fiber and 17.44 km of D_- fiber followed by an amplifier. The values of dispersion, effective core area, nonlinear index, and loss were 20.17 ps/nm-km, 106.7 μm^2, 1.7×10^{-20} m^2/W, and 0.19 dB/km for the D_+ fiber, and -40.8 ps/nm-km, 31.1 μm^2, 2.2×10^{-20} m^2/W, and 0.25 dB/km for the D_- fiber, respectively. The average map dispersion is -0.5 ps/nm-km, and the amount of pre- and post-compensation is 1028 and 1815 ps/nm respectively, which was optimized by maximizing the eye opening in the center of the bit slot. We used 35 ps raised-cosine pulses with a peak power of 5 mW and we launched nine co-polarized channels separated by 50 GHz, each carrying a 32-bit sequence. We verified that a further increase in the number of channels and number of bits per channel had a negligible effect on the system performance. The receiver included a 30 GHz super-Gaussian optical demultiplexer and a photodetector. We did not consider noise in the calculation of the time shift pdf.

Figure 1(a) shows the time shift pdfs obtained with the Gaussian approximation, the characteristic function method, and Monte Carlo simulations with importance sampling (IS) [6]. The excellent agreement between the pdfs obtained using the characteristic function method [6] and Monte Carlo simulations indicates that the simplifying assumption made in [7] that pulse collisions are additive is valid. By contrast, under the same assumption, the Gaussian approximation results in a poor fit of the true pdf. The Gaussian function closely matches the Monte Carlo histogram only near the top of the distribution, which is consistent with the central limit theorem.

3. Calculation of BER

In order to compute the pdf $p(I, t)$ of the received current I at the sampling time t due to both nonlinear signal distortions and noise, we must convolve the pdf p_T of the timing shift with the pdf due to noise $p_{\text{noise}}(I, t)$

of the received current that is obtained by propagating a single-channel signal through the system:

$$p(I,t) = \int p_{\text{noise}}(I, t-\tau)\, p_T(\tau) d\tau. \tag{3}$$

To compute $p_{\text{noise}}(I,t)$, we assume that the optical noise is additive white Gaussian noise (AWGN) at the entry to the receiver, and we take into account the actual pulse shapes and the shapes of the optical and electrical filter [11]. For the calculation of p_{noise}, and hence of $p(I,t)$ including the effect of noise, we used an 8-GHz electrical fifth-order Bessel filter and we set the optical signal-to noise ratio to 15 dB, calculated over a 25 GHz bandwidth.

In Fig. 1(b), we plot the BER and Q-factor as a function of the channel separation. We calculated the BER from (3) using the time shift pdfs obtained from the Gaussian approximation and from the characteristic function method. For each channel spacing, the total number of pump channels was chosen such that it filled the spectral range of 800 GHz around the center frequency of the target channel. A further increase in the number of channels had a negligible effect on the time shift pdf and the BER since the collision-induced time shift decreases quadratically with the channel spacing [9].

As we see in Fig. 1(b), the difference in the BERs computed with the two methods is less than an order of magnitude when ASE noise is included. The noise pdf is critical in producing this result. Its convolution with the time shift pdf leads to averaging that greatly reduces the difference between the BERs that is obtained by using the two different methods. However, we note that in the case of 25-GHz-spaced channels, the timing jitter leads to a very large value of the BER. At this point, the relative difference in the BER estimates obtained with the two methods is much more significant, and it may become critically important when applying forward error correction (FEC) techniques, since there is usually a sharp BER threshold above which FEC ceases to function well.

4. Conclusion

We have analyzed a method for calculating the BER in WDM-RZ systems impaired by the collision-induced timing jitter and ASE noise. The method is based on approximating the pdf of the collision-induced time shift with a Gaussian function. We have compared this approach with the previously reported characteristic function method and demonstrated that the two methods lead to significant differences in the time shift pdf but yield agreement that is within an order of magnitude when the ASE noise is taken into account. However, the difference in the predicted BER of nearly an order of magnitude can become important when the BER approaches the threshold at which FEC fails.

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