

MATH 2415 Calculus of Several Variables
Fall-2021

Skill Building Problems

To succeed in Calculus III you need to be competent at the following Calculus I,II and algebra topics: chain and product rules, integration by u -substitution and by parts, some basic integrals of trig functions, completing the square, solving polynomial equations. and local and absolute max/min problems. The problems below give you practice at these topics. Many of them are harder than we will encounter in Calculus III, but if you can solve these you will be in excellent shape!

1. Use the chain rule to find the derivatives of the following functions

(a) $y = \cos^3 x$

(b) $y = \cos(x^3)$

(c) $y = \tan\left(\frac{\pi}{3}\sqrt{x^2 + e^{3x-4}}\right)$

(d) $y = \sqrt{\ln(3x^2 + \sec^2 x)}$

(e) $y = \ln(\sqrt{3x^2 + \tan x})$

2. Find the derivatives of the following functions

(a) $y = [\sin(2x) + x^2] [\ln(3x^2 + 4) + x^3]$

(b) $y = \frac{1 + e^{-2x}}{2 - 3e^{-4x}}$

3. Evaluate the following definite integrals

(a) $\int_1^2 (x^3 - 4)^4 x^2 dx$

(b) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} e^{\sec(2x)} \sec(2x) \tan(2x) dx$

(c) $\int_2^7 \frac{x-1}{\sqrt{x+2}} dx$

(d) $\int_e^{e^2} \frac{1}{x(\ln x)^2} dx$

(e) $\int_{\ln 2}^{\ln 3} \frac{1}{2e^{-x} + 1} dx$

4. Evaluate the following integrals using the integration by parts

(a) $\int (x^2 - 3) e^{3x} dx$

(b) $\int (x^2 - 3) \sin(3x) dx$

(c) $\int (3x + 2) \ln(2x) dx$

(d) $\int e^{2x} \cos(3x) dx$

5. Find the following indefinite integrals

(a) $\int \sin^2 x dx$

(b) $\int \cos^2 x dx$

(c) $\int \sin^4 x dx$

(d) $\int \cos^4 x dx$

(e) $\int \sin^3 x \cos^2 x dx$

(f) $\int \sin^2 x \cos^3 x dx$

(g) $\int \sin^4 x \cos^2 x dx$

6. Verify the identity: $x^2 + mx = \left(x + \frac{m}{2}\right)^2 - \frac{m^2}{4}$ and use it to complete the squares in the following expressions.

(a) $x^2 + 6x$

(b) $x^2 - 6x$

(c) $x^2 + 6x + 17$

(d) $x^2 - 7x$

(e) $x^2 - 7x + 17$

(f) $3x^2 + 6x + 7$

(g) $7 + 6x - x^2$

7. Solve the following equations

(a) $(x + 1)(2x - 3) = 0$

(b) $(x + 1)(2x - 3) = 3$

(c) $2x^2 - x - 3 = 0$

(d) $12 - 2x - 2x^2 = 0$

(e) $6x - 5x^2 = -8$

8. Find all relative extrema of the following functions (Use the second derivative test when applicable).

(a) $f(x) = \frac{x^2}{2} - 4 \ln|x|$

(b) $f(x) = (x - 1)^3(x - 2)^2$

(c) $f(x) = (x - 1)^{\frac{1}{3}}(x - 2)^2$

9. Find the absolute extrema of the following functions on the given closed intervals

(a) $3x^{\frac{2}{3}} - 2x$ on $[-1, 1]$

(b) $\sin^2 x + \cos x$ on $[0, 2\pi]$

10. A capsule is formed by adjoining two hemispheres to the ends of a right circular cylinder. If the volume of the capsule has to be 84 cu.mm., find the radius of the cylinder that minimizes the surface area of the capsule. Also, find the minimum surface area.