Math 2415

Friday Problem Session on 12.5 (Planes) and 15.7

Recall the following definitions:

(i) A **vector parametrization** of the line through the endpoint of the vector $\mathbf{a}$ in the direction of the vector $\mathbf{b}$ is given by $\mathbf{r}(t) = \mathbf{a} + t\mathbf{b}$, where $t \in \mathbf{R}$.

(ii) A **scalar parametrization** of the line in (i) is

$$x = a_1 + tb_1$$
$$y = a_2 + tb_2$$
$$z = a_3 + tb_3$$

where $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$.

(iii) A **level set equation** of a plane is an equation of the form $ax + by + cz = d$, where a, b, c, d are real numbers.

(iv) A **parametrization** of a plane through the endpoint of the vector $\mathbf{u}$ that contains the vectors $\mathbf{v}$ and $\mathbf{w}$ is of the form $\mathbf{r}(s, t) = \mathbf{u} + s\mathbf{v} + t\mathbf{w}$, where $s, t \in \mathbf{R}$.

For each problem start by drawing a schematic diagram that illustrates the geometrical relationships between the various points, lines, vectors, planes in the problem. Use your diagram to help you set up equations that will help you solve the problem.

Aim to do at least problems 1-14.

1. 12.5.24. Once you have found the level set equation, convert it to a graph (eg $z = f(x, y)$) and to a parametrization
2. 12.5.26. Also find a parametrization of this plane.
3. (a) 12.5.30. (b) Is there a plane that contains the line in 12.5.30 and is parallel to the plane $5x + 2y + 2z = 1$? Explain!!
4. 12.5.41
5. 12.5.45
6. Find a parametrization of the plane that contains both the point $(2, 4, 6)$ and the line $x = 7 - 3t$, $y = 3 + 4t$, $z = 5 + 2t$.
7. Find a parametrization for the line of intersection of the planes $3x - 6y - 2z = 3$ and $2x + y - 2z = 2$.
8. 15.7.1a
9. 15.7.3a
10. 15.8.1

11. 15.8.3b
12. 15.7.7
13. 15.8.7
14. 12.5.27
15. In most cases, the intersection of two lines in $\mathbf{R}^2$ is a point. In most cases, what can you say about the following situations:
    (a) The intersection of two lines in $\mathbf{R}^3$
    (b) The intersection of two planes in $\mathbf{R}^3$
    (c) The intersection of a line and a plane in $\mathbf{R}^3$
16. Consider the line $\mathbf{r}(t) = (1+2t, -1-t, 3t)$. Find the point of intersection of this line with the xz-plane. Does this line intersect the y-axis?
17. 15.7.5
18. 15.8.9