Math 2415

Friday Problem Session on 12.6, 15.7, 15.8, 13.1

1. In this problem we will graph the surface $y = x^2$ in $\mathbf{R}^3$. In $\mathbb{R}^2$ the equation $y = x^2$ represents a curve in the plane. However, the set of points (x, y, z) in $\mathbf{R}^3$ for which $y = x^2$ represents a surface in space. To graph this surface, first sketch the curve in the plane, and then pull it out along the z-axis to form a surface. What would you need to do to graph the surface $x^2 + 4z^2 = 1$?

2. In this problem we will graph the surface $z^2 - x^2 - y^2 = 4$ in space. First check that when you convert the equation $z^2 - x^2 - y^2 = 4$ to cylindrical coordinates you get $z^2 - r^2 = 4$. Plot the curve $z^2 - r^2 = 4$ in the (r, z)-plane. Then, by rotating this curve about the z-axis, graph the surface $z^2 - x^2 - y^2 = 4$ in space.

3. 12.6.21 Solve this matching problem using appropriately chosen traces (slices).

   For this and the related problems below make sure you label the axes of your trace plots as well as labeling each traces by its k-value. Also include any asymptotes and the coordinates of any intercepts in the sketch of your trace curves. Transfer as much of this information as you can to the sketch of the surface in 3-dimensional space.

4. 12.6.11

5. 12.6.15

6. Let C be the curve in the xy-plane given by $x = 2\sin t$, $y = -\cos t$. Find an equation of the form $f(x, y) = 0$ for this curve. Sketch the curve. What is the starting point (where $t = 0$)? What direction does the parametrization go along the curve?

7. By drawing on the white board, describe the motion given by the parametrized curve $x = \cos t$, $y = -\cos t$ for $0 \leq t \leq 3\pi$.

8. 13.1.14. Hint: Find a plane and a cylinder on which this curve lies.

9. 13.1.15

10. 13.1.21

11. 13.1.23

12. 13.1.27

13. 12.6.7

14. 12.6.23 Solve this matching problem using appropriately chosen traces (slices). Hint: Convert to cylindrical coordinates.

15. 12.6.25 Solve this matching problem using appropriately chosen traces (slices).