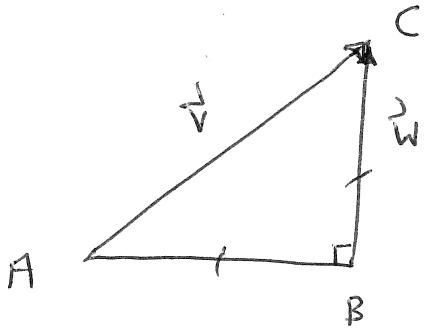
Math 2415
Homework on 12.4

1. Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$ and $c = \mathbf{j} + 2\mathbf{k}$.
   (a) Compute $\mathbf{a} \times \mathbf{b}$.
   (b) Find the length of $\mathbf{a}$ and a unit vector in the direction of $\mathbf{a}$.
   (c) Find a vector that is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.
   (d) Calculate the area of the parallelogram determined by the vectors $\mathbf{a}$ and $\mathbf{b}$.

2. The triangle ABC in the figure below is an isoceles triangle for which the length of the hypotenuse is 1. Calculate $\mathbf{v} \times \mathbf{w}$.



3. Let $P = (2, 0, 1)$, $Q = (3, 1, 0)$ and $R = (4, 3, 5)$.
   (a) Find the area of the triangle with vertices P, Q, and R.
   (b) Calculate a unit vector that is perpendicular to this triangle

4. Find the volume of the paralleliped with sides $\mathbf{i}$, $3\mathbf{j} - \mathbf{k}$, and $4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

5. Find nonzero vectors, $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$, so that $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, but $\mathbf{b} \neq \mathbf{c}$.

6. Find the area of the parallelogram with vertices $(10, 7, 13)$, $(1, 2, 3)$, $(4, 1, 7)$, $(7, 8, 9)$.