

# Math 2415

## Homework on 12.5

Recall the following definitions:

- (i) A **vector parametrization** of the line through the endpoint of the vector  $\mathbf{a}$  in the direction of the vector  $\mathbf{b}$  is given by  $\mathbf{r}(t) = \mathbf{a} + t\mathbf{b}$ , where  $t \in \mathbf{R}$ .
- (ii) A **scalar parametrization** of the line in (i) is

$$x = a_1 + tb_1$$

$$y = a_2 + tb_2$$

$$z = a_3 + tb_3$$

where  $\mathbf{a} = (a_1, a_2, a_3)$  and  $\mathbf{b} = (b_1, b_2, b_3)$ .

- (iii) A **level set equation** of a plane is an equation of the form  $ax + by + cz = d$ , where  $a, b, c, d$  are real numbers.
- (iv) A **parametrization** of a plane through the endpoint of the vector  $\mathbf{u}$  that contains the vectors  $\mathbf{v}$  and  $\mathbf{w}$  is of the form  $\mathbf{r}(s, t) = \mathbf{u} + s\mathbf{v} + t\mathbf{w}$ , where  $s, t \in \mathbf{R}$ . (Here we are assuming that  $\mathbf{v}$  and  $\mathbf{w}$  are not parallel.)

**For each problem start by drawing a schematic diagram that illustrates the geometrical relationships between the various points, lines, vectors, planes in the problem. Use your diagram to help you set up equations that will help you solve the problem.**

1. Find the level set equation of the plane through the point  $(1, 2, 3)$  parallel to the plane  $3x - 5y + 7z = 8$ .
2. Find the level set equation and a parametrization of the plane through the points  $(1, 0, -1)$ ,  $(3, 3, 2)$ , and  $(4, 5, -1)$ .
3. Does the line  $x = 3 + 2t$ ,  $y = 6 - 5t$ ,  $z = 2 + 3t$  intersect the plane  $3x + 2y - 4z = 1$ ?
4. Find the equation of the plane that contains the point  $(1, 2, 3)$  and is perpendicular to the line  $\mathbf{r}(t) = (4 - t, 5 + 6t, 7 - 2t)$ .
5. Find a parametrization of the line through the point  $(1, 2, -4)$  that is perpendicular to the plane  $x + 2y + 4z = 8$ .
6. Find a non-zero vector that is parallel to both of the planes  $x + y + 2z = 5$  and  $y - 6z = 7$ .
7. Find an equation of the plane that passes through the point  $(1, 2, 1)$  and contains the line of intersection of the planes  $x + y - z = 2$  and  $2x - 3y + 2z = 1$ .

8. Let  $P = (2, 0, 1)$ ,  $Q = (3, 1, 0)$  and  $R = (4, 3, 5)$ .
- (a) Find the level set equation of the plane,  $\mathcal{P}$ , containing  $P$ ,  $Q$ , and  $R$ .
  - (b) Find a parametrization of the plane,  $\mathcal{P}$ , containing  $P$ ,  $Q$ , and  $R$ .
  - (c) Let  $\mathcal{L}$  be the line passing through the point  $(-1, 0, 2)$  that is parallel to the vector  $(1, 2, 3)$ . Find the point of intersection of this line with the plane,  $\mathcal{P}$ .
  - (d) Is the line through the points  $(2, 4, -5)$  and  $(0, 1, 3)$  parallel to the plane,  $\mathcal{P}$ ? Explain why or why not.