Math 2415
Homework on 14.5

Let $z = f(x, y)$ be a function on the plane and let $(x, y) = \mathbf{r}(t)$ be a curve in the plane. The composition $z = g(t) = f(\mathbf{r}(t))$ is a scalar-valued function of one variable. The function g is called the **restriction** of f to the curve $\mathbf{r}$, since it just gives us the values of f along the curve $\mathbf{r}$. In this context, the **Chain Rule for Functions on Curves** states that $g'(t) = \nabla f(\mathbf{r}(t)) \cdot \mathbf{r}'(t)$.

1. Let $(x, y) = \mathbf{r}(t) = (t, \sqrt{t})$ be a curve in the plane and let $z = f(x, y) = \exp(-x^2 - y^2)$ a function on the plane. Let $g(t) = f(\mathbf{r}(t))$ be the restriction of f to the curve $\mathbf{r}$. Use the Chain Rule for Functions on Curves to calculate $g'(4)$.

2. Let $z = f(x, y) = x^2 \cos y$ where $x = x(t) = \sin t$ and $y = y(t) = t^3$.

   (a) Form the composition $g(t) = f(x(t), y(t))$ and then use the single variable chain rule to calculate $g'(t)$.

   (b) Use the Chain Rule for Functions on Curves to calculate $g'(t)$.

3. Problem 4 from http://mathquest.carroll.edu/libraries/MVC.student.14.06.pdf

4. Suppose that $z = f(x, y) = \sin(2x + 3y)$ where $x = x(t)$ and $y = y(t)$. If $x(0) = \pi/4$, $y(0) = \pi/3$, $x'(0) = -2$, and $y'(0) = 4$, find $\frac{dz}{dt}$ at $t = 0$.

5. Draw a tree diagram and write a Chain Rule formula for each derivative.

   (a) $\frac{dz}{dt}$ for $z = f(u, v, w)$ with $u = g(t)$, $v = h(t)$, and $w = k(t)$.

   (b) $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ for $w = h(x, y, z)$ with $x = f(u, v)$, $y = g(u, v)$ and $z = k(u, v)$.

6. Problem 5 from http://mathquest.carroll.edu/libraries/MVC.student.14.06.pdf