Math 2415
Homework on 15.9

1. Solve for x and y in terms of u and v and then compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.

   (a) $u = x^2 - y^2$, $v = x + y$

2. Let R be the parallelogram bounded by the lines $3x+2y = 1$, $3x+2y = 5$, $2x-4y = -2$, $2x - 4y = 1$.

   (a) Use the change of variables $u = 3x + 2y$, $v = 2x - 4y$ to find it's area $A = \iint_R 1\, dx\, dy$.

   (b) Check that you get the same answer by using the formula $A = |\mathbf{a} \times \mathbf{b}|$, where $\mathbf{a}$ and $\mathbf{b}$ are two vectors which together determine the paralellogram.

   (c) Calculate $\iint_R x\, dx\, dy$.

3. Let S be the unit square in the uv-plane with vertices $(0, 0)$, $(1, 0)$, $(0, 1)$ and $(1, 1)$ and let D be the circle $u^2 + v^2 = 1$ in the uv-plane. Find the images of S and D under the following transformations.

   (a) $x = 3u + 2v$, $y = 2u - 4v$

4. Evaluate $\iint_R (x + 2y)(y - 3x)\, dA$ where R is the parallelogram enclosed by the lines $x + 2y = -4$, $x + 2y = 3$, $y - 3x = -1$, $y - 3x = 5$.

5. Use elliptical coordinates $x = 3r \cos\theta$ and $y = 2r \sin\theta$ to find the volume bounded by the paraboloid $z = x^2 + y^2$, the plane $z = 0$ and the elliptical cylinder $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

6. Use the change of variables $u = y/x^2$, $v = x/y^2$ to find the area of the region in the first quadrant that is bounded by the curves $y = x^2$, $y = 3x^2$, $x = y^2$ and $x = 4y^2$.