Math 2415
Homework on 16.3

1. Show that the vector field $\mathbf{F}(x, y) = (1 - ye^{-x})\mathbf{i} + e^{-x}\mathbf{j}$ is conservative. Find a function f so that $\mathbf{F} = \nabla f$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $\mathbf{r}(t) = e^t\mathbf{i} + \sin t\mathbf{j}$, for $0 \leq t \leq \pi/2$.

2. Determine whether the vector field, $\mathbf{F}$, is conservative or not. If it is, find a potential function, f, so that $\mathbf{F} = \nabla f$.

   (a) $\mathbf{F}(x, y) = 7e^x \sin y\,\mathbf{i} + 7e^x \cos y\,\mathbf{j}$

   (b) $\mathbf{F}(x, y) = (3x^2 + 2y^2)\,\mathbf{i} + (4xy + 6y^2)\,\mathbf{j}$

3. Show that the given line integral is independent of path, then calculate the value of the line integral.

   (a) $\int_C (\sin y + y\cos x)\,dx + (\sin x + x\cos y)\,dy$, where C goes from $(\pi/2, \pi/2)$ to (π, π).

4. Let $\mathbf{F}(x, y) = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ be a constant vector field. Show that no work is done by $\mathbf{F}$ on a particle that goes once around a closed curve.