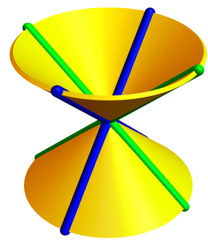
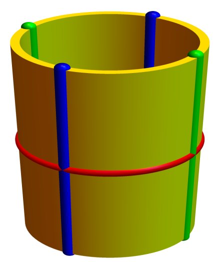
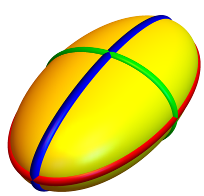
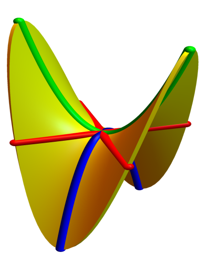
Math 2415
Homework on 16.6

1. In each of the following surface parametrizations, filling in the missing two entries. The highlighted curves are "grid curves" for the parametrization. Justify your work.

a)		The surface $\boxed{x^2+y^2=z^2}$ is parametrized by $\vec{r}(\theta,z)=\langle \ldots\ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots\ldots, z\rangle$
b)		The surface $\boxed{x^2+y^2=1}$ is parametrized by $\vec{r}(\theta,z)=\langle \ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots, z\rangle$
c)		The surface $\boxed{2(x-1)^2+(y-5)^2+4z^2=1}$ is parametrized by $\vec{r}(\theta,\phi)=\langle \ldots\ldots\ldots\ldots, \ldots\ldots\ldots\ldots, \cos(\phi)/2\rangle$
d)		The surface $\boxed{x^2-y^2=z}$ is parametrized by $\vec{r}(x,y)=\langle \ldots\ldots\ldots, \ldots\ldots\ldots, x^2-y^2\rangle$

2. (a) Identify the surface with parametrization

$$x=3\cos\theta\,\sin\phi \qquad y=3\sin\theta\,\sin\phi \qquad z=\cos\phi$$

where $0\le\theta\le 2\pi$ and $0\le\phi\le\pi$. **Hint:** Find an equation of the form $F(x,y,z)=0$ for this surface by eliminating θ and ϕ from the equations above.

(b) Calculate a parametrization for the tangent plane to the surface at $(\theta, \phi) = (\pi/3, \pi/4)$.

3. Let S be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = u \cos v \, \mathbf{i} + u \sin v \, \mathbf{j} + u \, \mathbf{k} \qquad u \geq 0, \quad 0 \leq v \leq 2\pi.$$

   (a) Show that S is a cone. **Hint:** Find an equation of the form $F(x, y, z) = 0$ for this surface by eliminating u and v from the equations for x, y, and z above.

   (b) Sketch the cone, together with the "grid" curves on the cone where (a) $u = 2$ and (b) $v = \pi/4$.

   (c) Find a parametrization of the tangent plane to the cone at the point where $(u, v) = (2, \pi/4)$. Add this tangent plane to your sketch.

4. Find a parametrization for each of the following surfaces. [Note: There are many correct answers!] Show how you arrived at your answer. **Hint:** It is often helpful to construct your parametrization using (a) cylindrical coordinates, (b) spherical coordinates, or (c) by using a parametrization such as $(x, y, z) = (u, v, f(u, v))$ for the surface $z = f(x, y)$.

   (a) The portion of the sphere $x^2 + y^2 + z^2 = 9$ that is above the cone $z = \sqrt{x^2 + y^2}$.

   (b) The portion of the cylinder $y^2 + z^2 = 9$ between the planes $x = 0$ and $x = 3$.