MATH 2415 Calculus of Several Variables

Fall-2019

PLTLWeek#10(Sec 15.1, 15.2)

---

1. Compute the definite integral $\iint\limits_R f(x,y)\,dA$

   (a) $f(x,y)=3;\quad R=\{(x,y):-2\le x\le 2, 0\le y\le 3\}$

   (b) $f(x,y)=4-x-y;\ R=[0,2]\times[0,2]$

   (c) $f(x,y)=e^{-y};\ R=[0,6]\times[0,\ln 2]$

   (d) $f(x,y)=\sin x\sin y;\ R=[0,\pi]\times[0,\pi]$

2. Compute the average value of the following functions over the region R

   (a) $f(x,y)=3;\quad R=\{(x,y):-2\le x\le 2, 0\le y\le 3\}$

   (b) $f(x,y)=4-x-y;\ R=[0,2]\times[0,2]$

   (c) $f(x,y)=e^{-y};\ R=[0,6]\times[0,\ln 2]$

   (d) $f(x,y)=\sin x\sin y;\ R=[0,\pi]\times[0,\pi]$

3. Find the volume of the solid beneath the surface $z=f(x,y)$ and above the rectangle R on the $xy-$plane.

   (a) $f(x,y)=3;\quad R=\{(x,y):-2\le x\le 2, 0\le y\le 3\}$

   (b) $f(x,y)=4-x-y;\ R=[0,2]\times[0,2]$

   (c) $f(x,y)=e^{-y};\ R=[0,6]\times[0,\ln 2]$

   (d) $f(x,y)=\sin x\sin y;\ R=[0,\pi]\times[0,\pi]$

4. Calculate the following iterated integrals

   (a) $\displaystyle\int_1^4\int_0^2 \left(6x^2y-2x\right)\,dy\,dx$

   (b) $\displaystyle\int_{-3}^3\int_0^{\frac{\pi}{2}} \left(y+y^2\cos x\right)\,dy\,dx$

   (c) $\displaystyle\int_1^3\int_1^5 \frac{\ln y}{xy}\,dy\,dx$

   (d) $\displaystyle\int_1^4\int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right)\,dy\,dx$

   (e) $\displaystyle\int_1^4\int_0^2 xe^{2xy}\,dy\,dx$

   (f) $\displaystyle\int_1^4\int_0^2 xe^{2xy}\,dx\,dy$

   (g) $\displaystyle\int_1^4\int_0^2 ye^{2xy}\,dy\,dx$

   (h) $\displaystyle\int_0^1\int_0^2 ye^{x-y}\,dy\,dx$

5. Calculate the double integrals

   (a) $\displaystyle\iint_R (4-x-2y)\,dA;\ R=[0,1]\times[0,1]$

   (b) $\displaystyle\iint_R \left(2-x^2-y^2\right)\,dA;\ R=[0,1]\times[0,1]$

(c) $\displaystyle\iint_R \frac{x}{1+xy}\,dA;\ R=[0,1]\times[0,1]$

(d) $\displaystyle\iint_R 6xe^{3y}\,dA;\ R=[0,\ln 2]\times[0,1]$

(e) $\displaystyle\iint_R x\cos(xy)\,dA;\ R=\left[0,\frac{\pi}{2}\right]\times[0,1]$

(f) $\displaystyle\iint_R 6x^5e^{x^3y}\,dA;\ R=[0,2]\times[0,2]$

6. Evaluate the following iterated integrals

(a) $\displaystyle\int_1^5\int_0^x (3x-4y)\,dy\,dx$

(b) $\displaystyle\int_1^5\int_0^{e^x} (3x-4y)\,dy\,dx$

(c) $\displaystyle\int_0^1\int_0^y x^2e^{y^4+1}\,dx\,dy$

7. Evaluate the following double integrals

(a) $\displaystyle\iint_D \frac{y}{x^2+1}\,dA,\ D=\{(x,y):0\le x\le 4, 0\le y\le\sqrt{x}\}$

(b) $\displaystyle\iint_D e^{-x^2}\,dA,\ D=\{(x,y):0\le x\le 4, 0\le y\le x\}$

(c) $\displaystyle\iint_D x\sqrt{y^2-x^2}\,dA,\ D=\{(x,y):0\le y\le 4, 0\le x\le y\}$

(d) $\displaystyle\iint_D \left(y^2+2\right)\,dA$, where D is the triangular region with vertices $(0,1),(1,2),(4,1)$

(e) $\displaystyle\iint_D \left(x^2+2y\right)\,dA$, where D is bounded by the graph of $y=x,\ y=x^3, x\ge 0$

8. Reverse the order of integration to evaluate the following double integrals

(a) $\displaystyle\int_0^1\int_{4y}^4 e^{x^2}\,dx\,dy$

(b) $\displaystyle\int_0^2\int_{\frac{y}{2}}^1 y\cos\left(x^3-1\right)\,dx\,dy$

(c) $\displaystyle\int_0^8\int_{\sqrt[3]{y}}^2 e^{x^4}\,dx\,dy$