MATH 2415 Calculus of Several Variables

Fall-2018

PLTLWeek#11(Sec 15.3, 15.6)

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1. Evaluate the following integrals by changing into polar coordinates

   (a) $\iint\limits_D \cos\left(\sqrt{x^2+y^2}\right)\, dA$, where D is the disk centered at origin and radius 3.

   (b) $\iint\limits_D \cos\left(\sqrt{x^2+y^2}\right)\, dA$, where D is the annular region $16 \le x^2+y^2 \le 36$.

   (c) $\int\limits_0^{\frac{1}{2}} \int\limits_{\sqrt{3}y}^{\sqrt{1-y^2}} xy^2\, dx\, dy$

2. Use polar coordinates to find the volume of the solid below the plane $2x+y+z=4$ and above the disk $x^2+y^2 \le 1$.

3. Use polar coordinates to find the volume of the solid below the plane $2x+y+z=14$ and above the region $D=\{(x,y) : 1 \le x^2+y^2 \le 4;\ y \ge 0\}$.

4. Use polar coordinates to find the average value of the function $f(x,y)=4-2x-y$ on the disk $x^2+y^2 \le 1$.

5. Use polar coordinates to find the average value of the function $f(x,y)=14-2x-y$ on the region $D=\{(x,y) : 1 \le x^2+y^2 \le 4;\ y \ge 0\}$.

6. Evaluate the following triple integrals

   (a) $\int\limits_{-2}^{2} \int\limits_{1}^{2} \int\limits_{1}^{e} \frac{xy^2}{z}\, dz\, dy\, dx$

   (b) $\int\limits_0^{\ln 4} \int\limits_0^{\ln 3} \int\limits_0^{\ln 2} e^{-x+y+z}\, dx\, dy\, dz$

   (c) $\int\limits_0^{\frac{\pi}{2}} \int\limits_0^{1} \int\limits_0^{\frac{\pi}{2}} \sin \pi x \cos y \sin 2z\, dy\, dx\, dz$

   (d) $\iiint\limits_E (xy+xz+yz)\, dV,\ E=\{(x,y,z) : -1 \le x \le 1,\ -2 \le y \le 2,\ -3 \le z \le 3\}$

   (e) $\iiint\limits_E xyze^{-x^2-y^2}\, dV,\ E=\{(x,y,z) : 0 \le x \le \sqrt{\ln 2},\ 0 \le y \le \sqrt{\ln 4},\ 0 \le z \le 1\}$

7. Evaluate the following integrals

   (a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} 2xz \, dz \, dy \, dx$

   (b) $\displaystyle\int_0^{\pi} \int_0^{\pi} \int_0^{\sin x} \sin y \, dz \, dx \, dy$

   (c) $\displaystyle\int_0^1 \int_y^{2y} \int_0^{2-x-y} xy \, dz \, dx \, dy$

   (d) $\displaystyle\int_1^{\ln 8} \int_1^{\sqrt{z}} \int_{\ln y}^{\ln 2y} e^{x+y^2-z} \, dx \, dy \, dz$

8. Find the volume of the solid bounded in the first octant by the plane $2x + 3y + 6z = 12$ and the coordinate planes.

9. Find the volume of the solid bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2+y^2+z^2 = 8$

10. Find the volume of the solid bounded by $x = 0,\ x = 1 - z^2,\ y = 0,\ z = 0$, and $z = 1 - y$

11. Rewrite the triple integral $\displaystyle\int_0^5 \int_{-1}^0 \int_0^{4x+4} dy \, dx \, dz$ in the order $dz \, dx \, dy$ and evaluate it.