MATH 2415 Calculus of Several Variables

Fall-2019

PLTLWeek# 14(Sec 16.3, 16.4, 16.5)

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1. Let $f(x,y,z) = xy + xz + yz$ and C be the curve given by vector function $\mathbf{r}(t) = t\mathbf{i} + 2t\mathbf{j} + 3t\mathbf{k};\ 0 \le t \le 4$, evaluate $\displaystyle\int_C \nabla f \cdot d\mathbf{r}$

2. Given vector field $\mathbf{F}(x,y) = (1+xy)e^{xy}\mathbf{i} + x^2e^{xy}\mathbf{j}$

   (i) Verify that $\mathbf{F}$ is conservative

   (ii) Find a potential function f. i.e. a function f such that $\mathbf{F} = \nabla f$.

   (iii) Evaluate the line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the parametric curve $\mathbf{r}(t) = \cos t\mathbf{i} + 2\sin t\mathbf{j};\ 0 \le t \le \frac{\pi}{2}$.

3. Show that $\displaystyle\int_C \sin y\, dx + (x\cos y - \sin y)\, dy$ is independent of path. Evaluate the integral for any path C from $(2,0)$ to $(1,\pi)$

4. Use Green's Theorem to evaluate $\displaystyle\oint_C xy\, dx + x^2y^3\, dy$ where C is the positively oriented triangle with vertices $(0,0), (1,0)$ and $(1,2)$

5. Use Green's Theorem to evaluate $\displaystyle\oint_C \left(x^2 + y^2\right)\, dx + \left(x^2 - y^2\right)\, dy$ where C is the positively oriented triangle with vertices $(0,0), (2,1)$ and $(0,1)$

6. Use Green's Theorem to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \langle y\cos x - xy\sin x,\ xy + x\cos x\rangle$ and C is the triangle from $(0,0)$ to $(0,4)$ to $(2,0)$ to $(0,0)$

7. Use Green's Theorem to evaluate $\displaystyle\int_C \left(y + e^{\sqrt{x}}\right)\, dx + \left(2x + \cos y^2\right)\, dy$, where C is the positively oriented boundary of the region enclosed by parabolas $y = x^2$ and $x = y^2$.

8. Use Green's Theorem to find the area of a circle of radius 7

9. Use Green's Theorem to find the area of the ellipse $4x^2 + 9y^2 = 36$

10. Find the curl: $\boldsymbol{\nabla} \times \mathbf{F}$ for the given vector field $\mathbf{F}$

    (a) $\mathbf{F}(x,y,z) = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$

    (b) $\mathbf{F}(x,y,z) = xye^z\mathbf{i} + yze^x\mathbf{k}$

    (c) $\mathbf{F}(x,y,z) = \langle e^x \sin y,\ e^y \sin z,\ e^z \sin x\rangle$

11. Find the divergence $\boldsymbol{\nabla} \cdot \mathbf{F}$ for given vector field $\mathbf{F}$.

    (a) $\mathbf{F}(x,y,z) = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$

    (b) $\mathbf{F}(x,y,z) = xye^z\mathbf{i} + yze^x\mathbf{k}$

(c) $\mathbf{F}(x, y, z) = \langle e^x \sin y,\ e^y \sin z,\ e^z \sin x \rangle$

12. Determine whether or not the vector field $\mathbf{F}$ is conservative.

    (a) $\mathbf{F}(x, y, z) = xy^2z^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$

    (b) $\mathbf{F}(x, y, z) = xye^z\mathbf{i} + yze^x\mathbf{k}$

    (c) $\mathbf{F}(x, y, z) = z\cos y\mathbf{i} + xz\sin y\mathbf{j} + x\cos y\mathbf{k}$

13. Prove that any vector field of the form

$$\mathbf{F}(x, y, z) = f(x)\mathbf{i} + g(y)\mathbf{j} + h(z)\mathbf{k}, \text{ where } f(x), g(y), h(z) \text{ are differentiable,}$$

    is irrotational.

14. Prove that any vector field of the form

$$F(x, y, z) = f(y, z)\mathbf{i} + g(x, z)\mathbf{j} + h(x, y)\mathbf{k}, \text{ is incompressible.}$$