Math 2415

Paper Homework #1

1. **[12.2 & 12.3, Vectors and Dot Products]** Let C be the point on the line segment AB that is twice as far from A and it is from B, and let O denote the origin. Let $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$.

   (a) Make a sketch showing the relationships between all these points and vectors.

   (b) Express $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

   (c) Hence express $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

   (d) Hence find a formula for $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

   (e) Now also suppose that $\mathbf{a} \perp \mathbf{b}$ and $|\mathbf{b}| = 1$. Find the scalar projection of $\mathbf{c}$ onto $\mathbf{b}$. *Hint:* Your answer should be a number.

   (f) Calculate $\mathbf{c}$ in the special case that $\mathbf{a} = \mathbf{i}$ and $\mathbf{b} = \mathbf{j}$. Is your answer consistent with your answer to (e)?

2. **[12.3, Dot Products]** In this problem you will find two unit vectors that each make an angle of 30° with the vector $\mathbf{v} = (3, 4)$.

   (a) Make a sketch that explains why there are precisely two such vectors $\mathbf{u}$.

   (b) Let $\mathbf{u} = (a, b)$ be one such vector. Write down an equation in terms of a and b that encodes the fact that $|\mathbf{u}| = 1$. Write down an equation in terms of a and b that encodes the fact that the angle between $\mathbf{u}$ and $\mathbf{v}$ is 30°.

   (c) Now you have two simultaneous equations in two unknowns, a and b. Solve them by eliminating one of the variables to obtain a quadratic equation for the other.

   (d) Hence find the two unit vectors.

   (e) Check your answers are correct.