Math 2415

Paper Homework #12

1. **16.2, Line Integrals:**

   (a) Evaluate $\int_C f\,ds$ where C has parametrization $\mathbf{r}$ with $\mathbf{r}(t) = (t, t^2, 1)$ for $0 \leq t \leq 2$ and $f(x, y, z) = x$.

   (b) Evaluate $\int_C yz\,dy + xy\,dz$ where C is the curve given by $x = \sqrt{t}$, $y = t$, $z = t^2$ for $1 \leq t \leq 2$.

2. **16.3, Conservative Vectors and FTC for Line Integrals:** Consider the vector field

$$\mathbf{F}(x, y) \;=\; (1 - ye^{-x})\mathbf{i} + e^{-x}\mathbf{j}.$$

   (a) Show that $\mathbf{F}$ is a conservative vector field.

   (b) Find a function f so that $\mathbf{F} = \nabla f$.

   (c) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $\mathbf{r}(t) = e^t\mathbf{i} + \sin t\mathbf{j}$, for $0 \leq t \leq \pi/2$.

3. **16.4, Green's Theorem:** Use Green's Theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = (6y + x)\mathbf{i} + (y + 2x)\mathbf{j}$ and C is the circle $(x - 2)^2 + (y - 3)^2 = 4$ traversed clockwise.