Math 2415

Paper Homework #4

1. **[12.6: Cylinders and Quadric Surfaces]** For each part, first decide which of the three methods we discussed in the lecture and problem sections will *most easily* enable you to sketch the surface. These are the generalized cylinder method, the surface of revolution method, and the method of slices (traces) for general quadric surfaces. Then apply that method to sketch the surface. You will be graded on how well you demonstrate your understanding of how to apply the method rather than for how perfect your final sketch looks.

   (a) $x^2 = 4z + 8$

   (b) $x^2 - 4y^2 - 4z^2 = 16$

   (c) $y^2 = x^2 + 2z^2$

2. **[13.1: Parametrized Curves]** This problem concerns three curves that lie on cylinders.

   (a) Consider the curve, C, parametrized by $x = \sin t$, $y = \cos t$, $z = \cos 4t$ for $0 \leq t \leq \pi$.

      i. What algebraic calculation shows that C lies on a cylinder.

      ii. Sketch the shadow of C on the xy-plane.

      iii. Sketch z as a function of t.

      iv. Calculate the (x, y, z)-coordinates of C at times $t = 0$, $\pi/4$, $\pi/2$, $3\pi/4$, and π.

      v. Use the information in the first four parts to sketch the curve C in space.

   (b) Use a method from the Problem Section on 13.1 to parametrize the curve obtained by intersecting the cylinder $x^2 + y^2 = 1$ and the saddle surface $z = x^2 - y^2$. Imagine you roll a piece of paper once around the cylinder, draw the curve on the paper, and then unwrap the paper and lay it flat. Draw the curve you get on the flat paper. From your picture, you can see that this curve is the graph of a function $f : [0, 2\pi] \to \mathbb{R}$. What is the equation for f?

   (c) Parametrize the *pair of curves* obtained by intersecting the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$. What are these curves? Where do they intersect? *For fun, not credit:* There are 3D-printed models you can use to help visualize these Intersecting Cylinders.