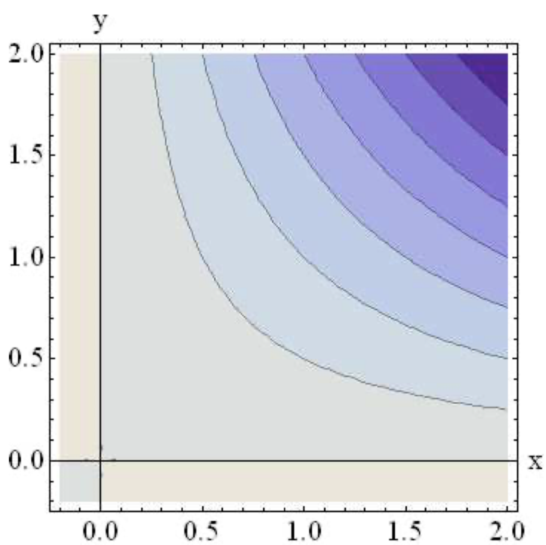
Math 2415
Paper Homework #6

1. **14.3, Partial Derivatives:**

   (a) **First Derivatives:**
   The plane $y = 2$ intersects the graph of $z = f(x, y) = xy^3 + 5x^2$ in a curve, C.
   i. Calculate $\frac{\partial f}{\partial x}$ at $(x, y) = (3, 2)$.
   ii. Find a parametrization of the tangent line to the curve C at the point where $x = 3$.

   (b) **Mixed Partials:** In this multiple choice problem provide a justification for your answer.

   In the contour plot below dark shades represent small values of the function and light shades represent large values of the function. What is the sign of the mixed partial derivative?



   (a) $f_{xy} > 0$
   (b) $f_{xy} < 0$
   (c) $f_{xy} \approx 0$
   (d) This cannot be determined from the figure.

   (c) **2nd Derivatives:** Verify that the following functions solve the wave equation, $u_{tt} = u_{xx}$.
   i. $u(x, t) = \cos(4x)\cos(4t)$
   ii. $u(x, t) = f(x - t) + f(x + t)$, where f is any differentiable function of one variable.

2. **14.4, Tangent Planes and Linear Approximation:**

   (a) Let $f(x, y) = x^2y^2 - x$.
   i. Find the equation for the tangent plane to the graph of f at $(2, 1, 2)$.
   ii. Use a linear approximation to find the approximate value of $f(1.9, 1.1)$.

(b) In this multiple choice problem provide a justification for your answer.

Suppose $f_x(3,4) = 5$, $f_y(3,4) = -2$, and $f(3,4) = 6$. Assuming the function is differentiable, what is the best estimate for $f(3.1, 3.9)$ using this information?

(a) 6.3
(b) 9
(c) 6
(d) 6.7

3. **14.5, Chain Rule:** Let $z = f(x,y)$ be a function on the plane and let $(x,y) = \mathbf{r}(t)$ be a curve in the plane. The composition

$$z = g(t) = f(\mathbf{r}(t)) = f(x(t), y(t))$$

is a scalar-valued function of one variable. The function g is called the **restriction** of f to the curve $\mathbf{r}$, since it just gives us the values of f along the curve $\mathbf{r}$. In this context, the **Chain Rule for Functions on Curves** states that

$$g'(t) \;=\; \nabla f(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \;=\; f_x(x(t), y(t))x'(t) \;+\; f_y(x(t), y(t))y'(t).$$

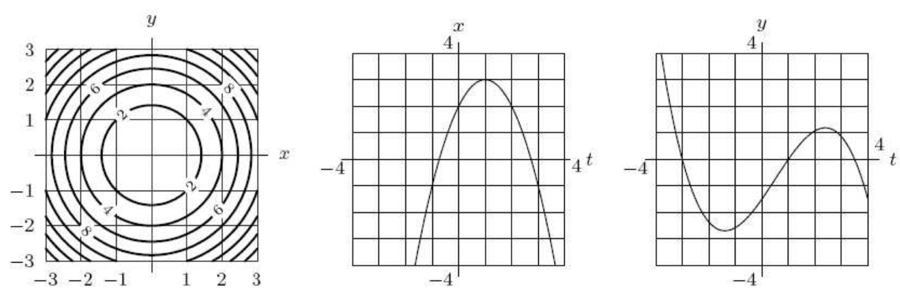
(a) Let $z = f(x,y) = x^2 \cos y$ where $(x,y) = \mathbf{r}(t) = (\sin t, t^3)$.

i. Form the composition $g(t) = f(x(t), y(t))$ and then use the single variable chain rule to calculate $g'(t)$.

ii. Use the Chain Rule for Functions on Curves to calculate $g'(t)$.

(b) In this multiple choice problem provide a justification for your answer.

The figures below show contours of $z = z(x,y)$, x as a function of t, and y as a function of t. Decide if $\left.\frac{dz}{dt}\right|_{t=2}$ is



(a) Positive
(b) Negative
(c) Approximately zero
(d) Can't tell without further information

(c) Draw a tree diagram and write a Chain Rule formula to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ for $w = h(x,y,z)$ with $x = f(u,v)$, $y = g(u,v)$ and $z = k(u,v)$.