MATH 2415 Calculus of Several Variables

Fall-2019

PLTL-Week#6 (Sec 14.3, 14.4, 16.6)

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1. Questions 5, 6, 7, 8 from the textbook (EX: 14.3, page 924)
2. Find all first and second order partial derivatives:

   (a) $f(x,y) = x^3 + 3x^2y - 3xy^2 - y^3$
   (b) $f(x,y) = 3x^2e^{x^2y}$
   (c) $f(x,y) = 3x^2y^2e^{x^2}$
   (d) $f(x,t) = x\sin(xt)$
   (e) $f(x,y) = \sqrt{3x^2 + 4xy - 4y^3}$
   (f) $f(r,\theta) = e^{-2r}\cos\theta$
   (g) $f(x,y) = \ln(3x - 4y)$
   (h) $u(s,t) = \sin\left(3s^2 - 4t^2\right)$

3. Prove that the following functions satisfy the Laplace's equation: $u_{xx} + u_{yy} = 0$

   (a) $u = 3x^2 + 4xy - 3y^2$
   (b) $u = e^{-2x}\sin 2y - e^{-3y}\cos 3x$
   (c) $u = \ln\sqrt{x^2 + y^2}$
   (d) $u = e^{-2x}\sin 2y$

4. Prove that the following functions satisfy the wave equation: $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$

   (a) $u = \sin(3x)\sin(3at)$
   (b) $u = (x - at)^6 + (x + at)^6$
   (c) $u = \sin(x - at) + \ln(x + at)$
   (d) $u = \cos(x + at)$

5. Find the equation tangent plane to the surface $f(x,y) = 3x^2e^{x^2y}$ at the point when $(x,y) = (1,0)$
6. Find the equation of tangent plane to the paraboloid $z = 2x^2 + 3y^2$ at the point $(1,1,5)$
7. Find the equation of the tangent plane to the surface $z = \ln\left(x^2 - y\right)$ when $(x,y) = (1,-1)$.
8. Find the equation of the tangent plane to the surface $z = x\sin(x + y)$ at the point $(-1,1,0)$.
9. Find the linearization $L(x,y)$ of the function $f(x,y) = 3x^2e^{x^2y}$ at $(1,0)$.
10. Find the linearization $L(x,y)$ of the function $z = 2x^2 + 3y^2$ at the point $(1,1)$.
11. Find the linearization $L(x,y)$ of the function $z = \ln\left(x^2 - y\right)$ when $(x,y) = (1,-1)$.
12. Find the linearization $L(x,y)$ of the function $z = x\sin(x + y)$ at $(-1,1)$.
13. The radius and height of a right circular cylinder are 4 in and 8 in with possible error in measurements up to 0.01 in and 0.02 in respectively. Use differentials to estimate the possible error in the calculated volume.
14. Find a parametric representation for the surface.

    (a) The plane through origin that contains the vectors $\mathbf{i} - \mathbf{j}$ and $\mathbf{j} - \mathbf{k}$.
    (b) The plane through $(1,1,-2)$ and containing the vectors $\langle 1,1,1\rangle$ and $\langle 3,2,1\rangle$.

(c) The part of the ellipsoid $x^2 + 4y^2 + 9z^2 = 36$ that lies on the first octant.

(d) The part of the cylinder $x^2 + z^2 = 9$ that lies above the $xy-$plane and between the planes $y = -4$ and $y = 4$.

(e) The part of the sphere $x^2 + y^2 + z^2 = 9$ that lies above the cone $z = \sqrt{x^2 + y^2}$.

15. Find an equation of the tangent plane to the following parametric surfaces

(a) $\mathbf{r}(u, v) = u^2\mathbf{i} + 2u \sin v\mathbf{j} + u \cos v\mathbf{k}$ when $(u, v) = (1, 0)$

(b) $\mathbf{r}(u, v) = \left(1 - u^2 - v^2\right)\mathbf{i} - v\mathbf{j} - u\mathbf{k}$; at point $(-1, -1, -1)$

(c) $\mathbf{r}(u, v) = u^2\mathbf{i} + uv\mathbf{j} + \frac{v^2}{2}\mathbf{k}$ when $(u, v) = (1, 2)$.