

# Math 2415

## Problem Section #13

*This week we will do problems from 16.5 and 16.6 as well as review for the Final Exam.*

*Based on past experience, about 50% of the points on the final exam will be on material from 15.3 onwards. In the previous problem session, posted the same set of exam review problems.*

### 16.5, Curl and Divergence

Let  $\mathbf{F}(x, y) = x^3\mathbf{i} + y^3\mathbf{j}$  be the velocity vector field of a fluid flowing in  $\mathbb{R}^2$ .

1. Calculate  $\nabla \cdot \mathbf{F}$ .
2. Calculate  $\nabla \times \mathbf{F}$ .
3. On average, is the fluid rotating clockwise, counter-clockwise, or not rotating at all about the point  $(1, 2)$ ? Why?
4. On average, is the fluid flowing in, out, or neither in or out, of a small disc centered at  $(1, 2)$ ? Why?

### 16.6, Parametrized Surfaces

1. **[Parts of Paper Hwk 16.6]:** Let  $S$  be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + u \mathbf{k} \quad u \geq 0, \quad 0 \leq v \leq 2\pi.$$

- (a) Show that  $S$  is a cone. **Hint:** Find an equation of the form  $F(x, y, z) = 0$  for this surface by eliminating  $u$  and  $v$  from the equations for  $x$ ,  $y$ , and  $z$  above.
  - (b) Find a parametrization of the tangent plane to the cone at the point where  $(u, v) = (2, \pi/4)$ .
2. (a) Write down the equation of the form  $F(x, y, z) = 0$  for the sphere of radius 2, center  $(1, 2, 3)$ .
  - (b) Show that

$$(x, y, z) = \mathbf{r}(\theta, \phi) = (1 + 2 \sin \phi \cos \theta, 2 + 2 \sin \phi \sin \theta, 3 + 2 \cos \phi)$$

is a parametrization of this sphere. **Hint:** Substitute the formulae for  $x$ ,  $y$ , and  $z$  in terms of  $\theta$  and  $\phi$  into the function  $F$  you obtained in (a) and simplify as much as you can. What does this calculation tell you about where each of the points  $(x, y, z) = \mathbf{r}(\theta, \phi)$  lie?

### Final Exam Review

Here are a long list of problems you could work on, many of which are exam questions from past semesters.

Also see Dr. Makhijani's [Final Exam Practice Problems](#), for which there are solutions [past exams webpage](#).

1. Stewart, 15.6.21
2. Stewart, 15.7.21
3. Stewart, 15.7.25 (a)
4. Stewart, 15.8.23
5. Stewart, 15.Review.30
6. Spring 2014 Final Exam # 8
7. Fall 2009 Exam II # 4
8. Fall 2014 Final Exam # 6
9. Spring 2014 Final Exam # 6
10. Spring 2004 Final: 1
11. Spring 2004 Final: 2
12. Spring 2004 Final: 6
13. Spring 2004 Final: 7 (Part d is on 16.6)
14. Spring 2008 Final: 1
15. Spring 2008 Final: 3
16. Spring 2008 Final: 4
17. Spring 2008 Final: 6
18. Spring 2019 Final: 10 (Based on 16.5)
19. Fall 2009 Final: 4 (Based on 16,.6)
20. Fall 2009 Final: 5
21. Fall 2009 Final: 6
22. Fall 2009 Final: 9