Math 2415

Problem Section #8

Work through the problems in order.

If you do not finish those for 14.8, there will be opportunity to finish them next week!

14.7B: Global Max/Min

1. Let D be the closed triangle with vertices $(0, 3)$, $(-3, 0)$, and $(3, 0)$. Find the absolute maximum and minimum of the function $f(x, y) = x^2 + y^2 - 2y$ on D.

2. Let D be the rectangle $[0, 4] \times [0, 2]$. Find the absolute maximum and minimum of the function $f(x, y) = 2x^2 + y^2 - 4x - 2y$ on D.

3. Let D be the quarter circle in the first quadrant with center $(0, 0)$ and radius 3. Find the absolute maximum and minimum of the function $f(x, y) = x^2 y$ on D.

14.8: The Method of Lagrange Multipliers, aka Constrained Optimization

Solution Strategy

For the problems from 14.8 you will use the method of Lagrange Multipliers to find the absolute maximum and minimum of a function $z = f(x, y)$ subject to a constraint $g(x, y) = k$. For these problems first solve the problem geometrically and then algebraically.

For the **geometric approach**, your task is to *visually spot those points,* (x_0, y_0)*, on the constraint curve which have the property that the tangent line to the constraint curve is equal to the tangent line to the level curve of* f *that goes through the point* (x_0, y_0). These points are called the critical points of the problem. By solving the problem geometrically/pictorially, you can often quickly determine the number of critical points as well as their approximate locations. In some cases it may also enable you to find the exact locations of the critical points exactly.

For the **algebraic approach** you need to solve three equations in three unknowns, (x, y, λ) for the critical points as we discussed in class. You should use the geometric solution to guide/check your algebraic calculation, and in particular to make sure you have found all the critical points.

Problems

1. Use the method of Lagrange multipliers to find the maximum and minimum of $z = f(x, y) = ye^x$ on the circle $x^2 + y^2 = 2$.

2. Use the method of Lagrange multipliers to find the maximum and minimum of $z = f(x, y) = xy^2$ on the circle $x^2 + y^2 = 1$.

3. (a) Use the method of Lagrange multipliers to find the maximum and minimum of $z = f(x, y) = x^2 + y^2$ on the circle, C, given by $(x - 1)^2 + y^2 = 1$.

   (b) Explain why the solutions to this problem gives the points on the circle, C, that are closest to and furthest from the origin.

   (c) There are two critical points for this Lagrange multipliers problem, one of which has $\lambda = 0$. Explain why the critical point with $\lambda = 0$ is also a critical point for the problem of finding the local maxima and minima of the function $z = f(x, y)$ in the entire plane, $\mathbb{R}^2$ (rather than on the circle, C).

4. In this problem you will find the absolute max and min of the function $z = f(x, y) = x^2 + y^2 - 4x - 4y$ on the closed disk $x^2 + y^2 \leq 9$.

   (a) Find critical points of f in the open disk $x^2 + y^2 < 9$.

   (b) Explain why the problem of finding the absolute maximum and minimum of the function $f(x, y) = x^2 + y^2 - 4x - 4y$ on the circle $x^2 + y^2 = 9$ is the same as the problem of finding the absolute maximum and minimum of the function $h(x, y) = 9 - 4x - 4y$ on the circle $x^2 + y^2 = 9$.

   (c) To solve the problem of optimizing the function $z = h(x, y)$ on the circle $x^2 + y^2 = 9$ you can just use the geometric approach outlined above. The symmetry of the problem will enable you to find the exact locations of the critical points!