Math 2415

Problem Section #9

Make sure you do some problems from each section.

Start by completing any problems below from 14.8 that you did not do last week.

14.8: The Method of Lagrange Multipliers, aka Constrained Optimization

Solution Strategy

For the problems from 14.8 you will use the method of Lagrange Multipliers to find the absolute maximum and minimum of a function $z = f(x, y)$ subject to a constraint $g(x, y) = k$. For these problems first solve the problem geometrically and then algebraically.

For the **geometric approach**, your task is to *visually spot those points,* (x_0, y_0)*, on the constraint curve which have the property that the tangent line to the constraint curve is equal to the tangent line to the level curve of* f *that goes through the point* (x_0, y_0). These points are called the critical points of the problem. By solving the problem geometrically/pictorially, you can often quickly determine the number of critical points as well as their approximate locations. In some cases it may also enable you to find the exact locations of the critical points exactly.

For the **algebraic approach** you need to solve three equations in three unknowns, (x, y, λ) for the critical points as we discussed in class. You should use the geometric solution to guide/check your algebraic calculation, and in particular to make sure you have found all the critical points.

Problems

1. Use the method of Lagrange multipliers to find the maximum and minimum of $z = f(x, y) = ye^x$ on the circle $x^2 + y^2 = 2$.

2. Use the method of Lagrange multipliers to find the maximum and minimum of $z = f(x, y) = xy^2$ on the circle $x^2 + y^2 = 1$.

3. (a) Use the method of Lagrange multipliers to find the maximum and minimum of $z = f(x, y) = x^2 + y^2$ on the circle, C, given by $(x - 1)^2 + y^2 = 1$.

   (b) Explain why the solutions to this problem gives the points on the circle, C, that are closest to and furthest from the origin.

   (c) There are two critical points for this Lagrange multipliers problem, one of which has $\lambda = 0$. Explain why the critical point with $\lambda = 0$ is also a critical point for the problem of finding the local maxima and minima of the function $z = f(x, y)$ in the entire plane, $\mathbb{R}^2$ (rather than on the circle, C).

4. In this problem you will find the absolute max and min of the function $z = f(x, y) = x^2 + y^2 - 4x - 4y$ on the closed disk $x^2 + y^2 \leq 9$.

   (a) Find critical points of f in the open disk $x^2 + y^2 < 9$.

   (b) Explain why the problem of finding the absolute maximum and minimum of the function $f(x, y) = x^2 + y^2 - 4x - 4y$ on the circle $x^2 + y^2 = 9$ is the same as the problem of finding the absolute maximum and minimum of the function $h(x, y) = 9 - 4x - 4y$ on the circle $x^2 + y^2 = 9$.

(c) To solve the problem of optimizing the function $z = h(x, y)$ on the circle $x^2 + y^2 = 9$ you can just use the geometric approach outlined above. The symmetry of the problem will enable you to find the exact locations of the critical points!

15.1, 15.2: Double Integrals (Rectangular Coordinates)

1. Evaluate the double integral, $\iint_R (2y + 3)\, dA$, where $R = [0, 3] \times [0, 2]$ by identifying it as the volume of a solid.

2. Calculate $\iint_R y e^{xy}\, dA$ where $R = [0, 1] \times [0, 2]$.

3. Sketch the solid enclosed by the paraboloid $z = 2 + x^2 + y^2$ and the planes $x = -1$, $x = +1$, $y = 0$, $y = 2$, and $z = 8$. Set up a double integral to calculate the volume of this solid and evaluate the integral.

4. Sketch a region that is Type I but not Type II.

5. Set up iterated integrals for both orders of integration for the integral $\iint_D y\, dA$, where D is bounded by $x = 0$, $y = x$ and $y = 3 - x$. In which order is easier to do the iterated integrals? Explain. Evaluate the integral this way.

6. Evaluate $\iint_D x^2\, dA$ where D is the triangular region with vertices $(0, 2)$, $(1, 3)$, and $(4, 0)$.

7. Evaluate the integral, $\int_{x=0}^{x=1} \int_{y=x^2}^{y=1} \sqrt{y} \sin(y)\, dy\, dx$ by reversing the order of integration.

8. Find the volume of the tetrahedron bounded by the coordinate planes and the plane $x + 2y + 3z = 6$.

9. Find the volume of the solid region under the plane $z = 4$, above the plane $z = x$, and between the parabolic cylinders $y = x^2$ and $y = 1 - x^2$.

10. **Review:** Fall 2016 Exam II, Questions 1,2,3.