

# Math 2415

## Paper Homework #1

*Some of the paper homework problems in this course are “scaffolded”. This means that we break a harder problem down into several manageable parts. When you are working on such problems it helps to work out how the answers to the earlier parts help you tackle the later parts. Schematic diagrams may help you solve each part and also connect different parts of the problem. A schematic diagram shows the relationships between the different nouns in the problem statement, but not in a completely realistic way.*

- [12.2 & 12.3, Vectors and Dot Products]** Let  $C$  be the point on the line segment  $AB$  that is twice as far from  $A$  and it is from  $B$ , and let  $O$  denote the origin. Let  $\mathbf{a} = \overrightarrow{OA}$ ,  $\mathbf{b} = \overrightarrow{OB}$ , and  $\mathbf{c} = \overrightarrow{OC}$ .
  - Make a sketch showing the relationships between all these points and vectors. Your sketch will help you solve the other parts of the problem.
  - Express  $\overrightarrow{AB}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Hence express  $\overrightarrow{AC}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Hence find a formula for  $\mathbf{c}$  in terms of  $\mathbf{a}$  and  $\mathbf{b}$ .
  - Now also suppose that  $\mathbf{a} \perp \mathbf{b}$  and  $|\mathbf{b}| = 1$ . Find the scalar projection of  $\mathbf{c}$  onto  $\mathbf{b}$ . *Hint:* Your answer should be a number.
  - Calculate  $\mathbf{c}$  in the special case that  $\mathbf{a} = \mathbf{i}$  and  $\mathbf{b} = \mathbf{j}$ . Is your answer consistent with your answer to (e)?
- [12.3, Dot Products]** In this problem you will find two unit vectors that each make an angle of  $30^\circ$  with the vector  $\mathbf{v} = (3, 4)$ .
  - Make a sketch that explains why there are precisely two such vectors  $\mathbf{u}$ .
  - Let  $\mathbf{u} = (a, b)$  be one such vector. Write down an equation in terms of  $a$  and  $b$  that encodes the fact that  $|\mathbf{u}| = 1$ . Write down an equation in terms of  $a$  and  $b$  that encodes the fact that the angle between  $\mathbf{u}$  and  $\mathbf{v}$  is  $30^\circ$ .
  - Now you have two simultaneous equations in two unknowns,  $a$  and  $b$ . Solve them by eliminating one of the variables to obtain a quadratic equation for the other.
  - Hence find the two unit vectors.
  - Check your answers are correct.