Math 2415

Paper Homework #12

1. **15.9, Change of Variables Theorem:**

   (a) Let R be the parallelogram bounded by the lines $3x+2y=1$, $3x+2y=5$, $2x-4y=-2$, $2x-4y=1$.

      i. Use the change of variables $u = 3x+2y$, $v = 2x-4y$ to find the area of R, ie find $A = \iint_R 1\, dx\, dy$.

      ii. Check that you get the same answer by using the formula $A = |\mathbf{a} \times \mathbf{b}|$, where $\mathbf{a}$ and $\mathbf{b}$ are two vectors which together determine the paralellogram.

      iii. Calculate $\iint_R x\, dx\, dy$.

   (b) Let S be the unit square in the uv-plane with vertices $(0,0)$, $(1,0)$, $(0,1)$ and $(1,1)$ and let D be the circle $u^2+v^2=1$ in the uv-plane. Find the images of S and D under the transformation $x = 3u+2v$, $y = 2u-4v$

   (c) Use elliptical coordinates $x = 3r\cos\theta$ and $y = 2r\sin\theta$ to find the volume bounded by the paraboloid $z = x^2+y^2$, the plane $z=0$ and the elliptical cylinder $\frac{x^2}{9}+\frac{y^2}{4}=1$.

   (d) Use the change of variables $u = y/x^2$, $v = x/y^2$ to find the area of the region in the first quadrant that is bounded by the curves $y = x^2$, $y = 3x^2$, $x = y^2$ and $x = 4y^2$.

Don't forget page 2!!

2. **16.1, Vector Fields**

(a) Match the vector fields $\mathbf{F}$ with the plots labeled A-D. Briefly explain your reasoning.

i. $\mathbf{F}(x, y, z) = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$

ii. $\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j}$

iii. $\mathbf{F}(x, y, z) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

iv. $\mathbf{F}(x, y, z) = -x\mathbf{i} - z\mathbf{k}$