

**Math 2415**  
**Paper Homework #12**

**1. 15.9, Change of Variables Theorem:**

- (a) Let  $R$  be the parallelogram bounded by the lines  $3x+2y = 1$ ,  $3x+2y = 5$ ,  $2x-4y = -2$ ,  $2x-4y = 1$ .
- i. Use the change of variables  $u = 3x + 2y$ ,  $v = 2x - 4y$  to find the area of  $R$ , ie find  $A = \iint_R 1 \, dx \, dy$ .
  - ii. Check that you get the same answer by using the formula  $A = |\mathbf{a} \times \mathbf{b}|$ , where  $\mathbf{a}$  and  $\mathbf{b}$  are two vectors which together determine the parallelogram.
  - iii. Calculate  $\iint_R x \, dx \, dy$ .
- (b) Let  $S$  be the unit square in the  $uv$ -plane with vertices  $(0, 0)$ ,  $(1, 0)$ ,  $(0, 1)$  and  $(1, 1)$  and let  $D$  be the circle  $u^2 + v^2 = 1$  in the  $uv$ -plane. Find the images of  $S$  and  $D$  under the transformation  $x = 3u + 2v$ ,  $y = 2u - 4v$
- (c) Use elliptical coordinates  $x = 3r \cos \theta$  and  $y = 2r \sin \theta$  to find the volume bounded by the paraboloid  $z = x^2 + y^2$ , the plane  $z = 0$  and the elliptical cylinder  $\frac{x^2}{9} + \frac{y^2}{4} = 1$ .
- (d) Use the change of variables  $u = y/x^2$ ,  $v = x/y^2$  to find the area of the region in the first quadrant that is bounded by the curves  $y = x^2$ ,  $y = 3x^2$ ,  $x = y^2$  and  $x = 4y^2$ .

*Don't forget page 2!!*

## 2. 16.1, Vector Fields

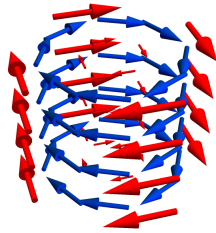
(a) Match the vector fields  $\mathbf{F}$  with the plots labeled A-D. Briefly explain your reasoning.

i.  $\mathbf{F}(x, y, z) = x\mathbf{i} + 2y\mathbf{j} + 3z\mathbf{k}$

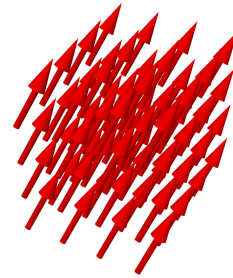
ii.  $\mathbf{F}(x, y, z) = y\mathbf{i} - x\mathbf{j}$

iii.  $\mathbf{F}(x, y, z) = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

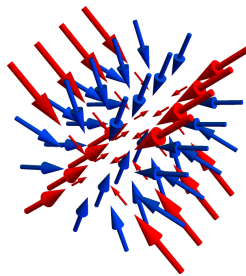
iv.  $\mathbf{F}(x, y, z) = -x\mathbf{i} - z\mathbf{k}$



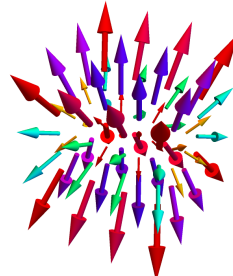
A



B



C



D