Math 2415

Paper Homework #4

1. **[12.6: Cylinders and Quadric Surfaces]** For each part, first decide which of the three methods we discussed in the lecture and problem sections will *most easily* enable you to sketch the surface. These are the generalized cylinder method, the surface of revolution method, and the method of slices (traces) for general quadric surfaces. Then apply that method to sketch the surface. You will be graded on how well you demonstrate your understanding of how to apply the method rather than for how perfect your final sketch looks.

   (a) $x^2 = 4z + 8$

   (b) $x^2 - 4y^2 - 4z^2 = 16$

   (c) $y^2 = x^2 + 2z^2$

2. **[13.1: Parametrized Curves]** This problem concerns three curves that lie on cylinders.

   (a) Consider the curve, C, parametrized by $x = \sin t$, $y = \cos t$, $z = \cos 4t$ for $0 \leq t \leq \pi$.

      i. What algebraic calculation shows that C lies on a cylinder.

      ii. Sketch the shadow of C on the xy-plane.

      iii. Sketch z as a function of t.

      iv. Calculate the (x, y, z)-coordinates of C at times $t = 0$, $\pi/4$, $\pi/2$, $3\pi/4$, and π.

      v. Use the information in the first four parts to sketch the curve C in space.

   (b) Use a method from the Problem Section on 13.1 to parametrize the curve obtained by intersecting the cylinder $x^2 + y^2 = 1$ and the saddle surface $z = x^2 - y^2$. Sketch the graph of $z = z(t)$ on a piece of paper so that the t-axis takes up one entire edge of the paper. Wrap the paper into a cylinder in such a way that the graph you sketched is the curve of intersection of the cylinder and saddle surface. Include your sketch of the graph of $z = z(t)$ in your homework solutions.

   (c) Parametrize the *pair of curves* obtained by intersecting the cylinders $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$. What are these curves? Where do they intersect? *For fun, not credit:* There are 3D-printed models you can use to help visualize these Intersecting Cylinders.