MATH 2415 Calculus of Several Variables

Fall-2019

PLTLWeek#12(Sec 15.7, 15.8, 15.9)

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1. Evaluate the following integrals by changing in to cylindrical coordinates

   (a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} 2xz\, dz\, dy\, dx$

   (b) $\displaystyle\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{4} dz\, dy\, dx$

   (c) $\displaystyle\int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^{\sqrt{x^2+y^2}} \left(x^2+y^2\right) dz\, dx\, dy$

2. Use cylindrical coordinates to calculate the volume of the solid bounded by the plane $z = 25$ and the paraboloid $z = x^2 + y^2$

3. Express the following solid regions using spherical coordinates.

   (a) Unit ball E

   (b) The solid between the spheres of radius 1 and 2 centered at origin.

   (c) The solid hemisphere $x^2 + y^2 + z^2 \leq 9,\ y \geq 0$

   (d) The portion of the unit ball $x^2 + y^2 + z^2 \leq 1$ that lies on the first octant.

   (e) $E = \{(x, y, z) : \sqrt{x^2+y^2} \leq z \leq \sqrt{2-x^2-y^2};\ 0 \leq y \leq \sqrt{1-x^2};\ 0 \leq x \leq 1\}$

   (f) $E = \{(x, y, z) : -\sqrt{25-x^2-y^2} \leq z \leq \sqrt{25-x^2-y^2};\ -\sqrt{25-y^2} \leq x \leq \sqrt{25-y^2};\ -5 \leq y \leq 5\}$

4. Evaluate the following integrals by changing into spherical coordinates

   (a) $\displaystyle\iiint_E \left(x^2+y^2+z^2\right)^{\frac{3}{2}} dV$, where E is the unit ball.

   (b) $\displaystyle\iiint_E e^{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}} dV$, where E is the unit ball.

   (c) $\displaystyle\iiint_E \left(x^2+y^2+z^2\right)^{\frac{5}{2}} dV$, where E is the unit ball.

   (d) $\displaystyle\iiint_E \frac{1}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}} dV$, where E is the solid between spheres of radius 1 and 2 centered at origin.

   (e) $\displaystyle\iiint_E y^2\, dV$, where E is the solid hemisphere $x^2 + y^2 + z^2 \leq 9,\ y \geq 0$

   (f) $\displaystyle\iiint_E x e^{x^2+y^2+z^2} dV$, where E is the portion of the unit ball $x^2 + y^2 + z^2 \leq 1$ that lies in the first octant.

(g) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy\,dz\,dy\,dx$

(h) $\displaystyle\int_{-5}^{5} \int_{-\sqrt{25-y^2}}^{\sqrt{25-y^2}} \int_{-\sqrt{25-x^2-y^2}}^{\sqrt{25-x^2-y^2}} \left(x^2z+y^2z+z^3\right)\,dz\,dx\,dy$

5. Use spherical coordinates to find the volume of the solid E that lies above the cone $z=\sqrt{x^2+y^2}$, below the sphere $x^2+y^2+z^2=2$, in the first octant.

6. Use spherical coordinates to find the average value of the function $f(x,y,z)=xy$ on

$$E=\{(x,y,z) : \sqrt{x^2+y^2} \le z \le \sqrt{2-x^2-y^2};\ 0 \le y \le \sqrt{1-x^2};\ 0 \le x \le 1\}$$

7. Use spherical coordinates to find the volume of the sphere of radius 5

8. Use spherical coordinates to find the average value of the function $f(x,y,z)=x^2z+y^2z+z^3$ on the sphere of radius 5 and centered at origin.

9. Use appropriate change of variables to evaluate $\displaystyle\iint_R \frac{x-2y}{3x-y}\,dA$, where R is the parallelogram enclosed by the lines $x-2y=0,\ x-2y=4,\ 3x-y=1$, and $3x-y=8$.

10. Use appropriate change of variables to evaluate $\displaystyle\int_1^2 \int_{\frac{1}{y}}^{y} \sqrt{\frac{y}{x}}\,e^{\sqrt{xy}}\,dx\,dy$

11. Use appropriate change of variables to evaluate $\displaystyle\iint_R \cos\left(\frac{x-y}{x+y}\right)\,dA$, where R is the trapezoidal region with vertices at $(1,0),(2,0),(0,2)$, and $(0,1)$.

12. Use the transformation $u=\frac{2x-y}{2},\ v=\frac{y}{2},\ w=\frac{z}{3}$ to evaluate $\displaystyle\int_0^3 \int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \left(\frac{2x-y}{2}-\frac{z}{3}\right)\,dx\,dy\,dz$

13. Let $T : uvw-$space $\to xyz-$space be a one to one transformation defined by

$$x=\rho\sin\phi\cos\theta,\ y=\rho\sin\phi\sin\theta,\ z=\rho\cos\phi.$$

Let T maps the region S in $uvw-$space onto region R in $xyz-$space, prove that

$$\iiint_R f(x,y,z)\,dV=\iiint_S f(\rho\sin\phi\cos\theta,\rho\sin\phi\sin\theta,\rho\cos\phi)\rho^2\sin\phi\,d\rho\,d\theta\,d\phi.$$