MATH 2415 Calculus of Several Variables

Fall-2019

PLTLWeek# 13(Sec 16.1, 16.2)

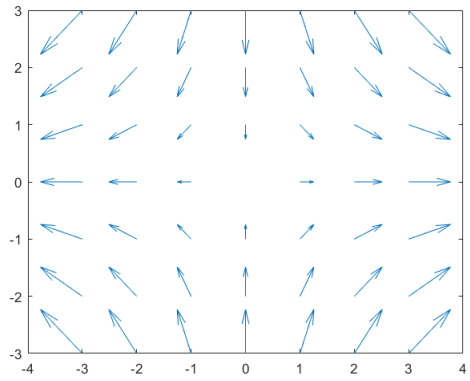
---

1. Sketch the following vector fields (at least 10 vectors)
   (a) $\mathbf{F}(x,y) = \langle x, y\rangle$.
   (b) $\mathbf{F}(x,y) = \left\langle \frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}} \right\rangle$
   (c) $\mathbf{F}(x,y) = \langle x, y-x\rangle$.
   (d) $\mathbf{F}(x,y) = \langle e^{-x}, 0\rangle$.

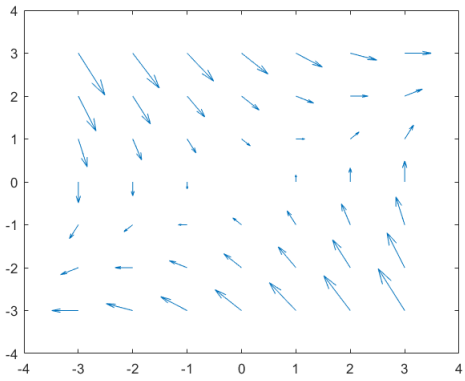
2. Match the vector fields and the graphs
   (a) $\mathbf{F}(x,y) = \langle x, -y\rangle$ (b) $\mathbf{F}(x,y) = \langle y,\ x-y\rangle$

   (c) $\mathbf{F}(x,y) = \langle y,\ y+2\rangle$ (d) $\mathbf{F}(x,y) = \langle \cos(x+y),\ x\rangle$

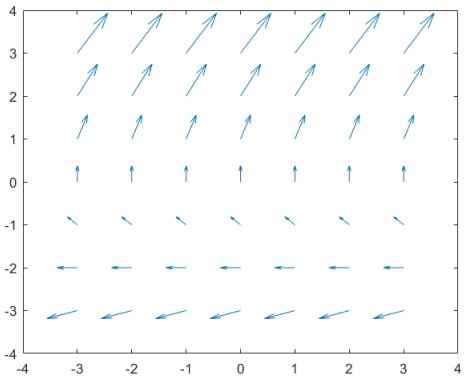
(I)



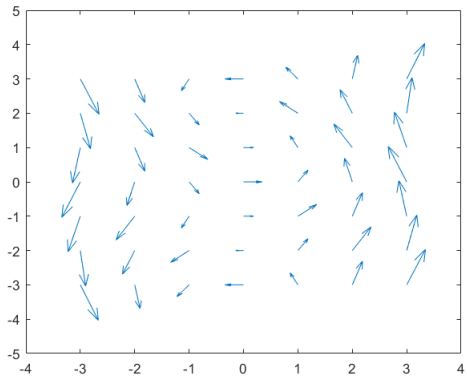
(II)



(III)



(IV)



3. Find the gradient vector fields for the following potential functions ϕ

   (a) $\phi(x, y) = x^2y - xy^2$

   (b) $\phi(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$

   (c) $\phi(x, y, z) = \ln\left(1 + x^2 + y^2 + z^2\right)$

   (d) $\phi(x, y, z) = \frac{GMm}{\sqrt{x^2+y^2+z^2}}$; G, M, m are constants.

4. Evaluate the following line integrals

   (a) $\int_C \left(x^2 + y^2\right) ds$; where C is the circle of radius 5 and center at origin.

   (b) $\int_C \left(x^2 + y^2\right) ds$; where C is the line segment from $(1, 1)$ to $(5, 5)$.

   (c) $\int_C xy\, ds$; where C is the portion of the ellipse $\frac{x^2}{4} + \frac{y^2}{16} = 1$ in the first quadrant.

   (d) $\int_C (2x - 3y)\, ds$; where C is the line segment from $(-1, 0)$ to $(0, 1)$ followed by the line segment from $(0, 1)$ to $(1, 0)$.

   (e) $\int_C (x - y + 2z)\, ds$; where C is the circle $\mathbf{r}(t) = \langle 1, 3\cos t, 3\sin t\rangle$; $0 \le t \le 2\pi$.

5. Find the average value of the function $f(x, y) = x^2 + y^2$ on the circle of radius 5 and center at origin.

6. Evaluate the line integrals $\int_C \mathbf{F} \cdot d\mathbf{r}$ of the vector fields over the parametric curve C.

   (a) $\mathbf{F} = \langle x, y\rangle$ where C is the parabola $\mathbf{r}(t) = \left\langle 4t, t^2\right\rangle$; $0 \le t \le 1$

   (b) $\mathbf{F} = \langle -y, x\rangle$ where C is the semicircle $\mathbf{r}(t) = \langle 4\cos t, 4\sin t\rangle$ above $x-$axis
   (i) clockwise direction; (ii) counterclockwise direction.

7. Find the work done by the force field $\mathbf{F} = \langle x, y\rangle$ on moving an object on the path consisting of the line segment from $(1, 2)$ to $(0, 0)$ followed by the line segment from $(0, 0)$ to $(0, 4)$.

8. Find the work done by the force field $\mathbf{F} = \frac{\langle x,y,z\rangle}{x^2+y^2+z^2}$ on moving an object on the line segment from $(1, 1, 1)$ to $(8, 4, 2)$.