# Math 2415

# Problem Section #11

*Do at least some problems from each section.*
*Recommended:15.2.4, 15.2,5, 15.3.2,15.3.4, 15.6 (all), 15.7 and 15.8 (all)*
**You will have the opportunity to do the remaining problems next week.**

## 15.2: Double Integrals (Rectangular Coordinates)

1. Sketch a region that is Type I but not Type II.
2. Set up iterated integrals for both orders of integration for the integral $\iint_D y\,dA$, where $D$ is bounded by $x = 0$, $y = x$ and $y = 3 - x$. In which order is easier to do the iterated integrals? Explain. Evaluate the integral this way.
3. Evaluate $\iint_D x^2\,dA$ where $D$ is the triangular region with vertices $(0, 2)$, $(1, 3)$, and $(4, 0)$.
4. Evaluate the integral, $\int_{x=0}^{x=1}\int_{y=x^2}^{y=1} \sqrt{y}\sin(y)\,dy\,dx$ by reversing the order of integration.
5. Find the volume of the tetrahedron bounded by the coordinate planes and the plane $x+2y+3z = 6$.
6. Find the volume of the solid region under the plane $z = 4$, above the plane $z = x$, and between the parabolic cylinders $y = x^2$ and $y = 1 - x^2$.
7. **Review:** Fall 2016 Exam II, Questions 1,2,3.

## 15.3, Double Integrals in Polar Coordinates

1. Evaluate $\iint_D e^{x^2+y^2}\,dA$, where $D$ is the region in the 1st quadrant between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.
2. Evaluate $\iint_D \cos(x^2+y^2)\,dA$, where $D$ is the region bounded by the semicircle $x = \sqrt{9 - y^2}$ and the $y$-axis.
3. Calculate the volume of the solid under $z = x^2 + y^2$ and above $x^2 + y^2 \leq 16$.
4. Calculate the volume of the solid below the plane $x + 2y + 3z = 6$ and above $x^2 + y^2 \leq 1$.
5. Evaluate the integral by converting to polar coordinates: $\int_0^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} (x + 2y)\,dy\,dx$.

## 15.6, Triple Integrals in Rectangular Coordinates

1. Sketch the region bounded by the following surfaces. Each pair of the surfaces intersects in a curve. Be sure to include these curves in your sketch. Then use a triple integral to calculate the volume of the solid.
   (a) $z = x^2 + y^2$, $x = 0$, $y = 0$, $z = 0$, $x + y = 1$.
   (b) $x = z^2$, $x = 8 - z^2$, $y = 1$, $y = 3$.
   (c) $y = z^2$, $y = z$, $x + y + z = 2$, $x = 0$

2. Evaluate $\iiint_E y\, dV$, where $E$ is the solid bounded by the surfaces $z = 2 - x^2$, $z = x^2 - 2$, $y = 0$ and $y = 1$.

3. Find the volume of the solid enclosed by the cylinder $z = x^2$ and the planes $y = 0$ and $y + z = 2$.

## 15.7 and 15.8, Triple Integrals in Cylindrical and Spherical Coordinates

1. Use cylindrical coordinates to find the volume of the solid that lies both within the cylinder $x^2 + y^2 = 3$ and the sphere $x^2 + y^2 + z^2 = 4$.

2. Let $E$ be the solid region in the first octant (*i.e.*, where $x \geq 0$, $y \geq 0$, $z \geq 0$) that is inside the cylinder $x^2 + y^2 = 1$ and below the plane $x + z = 1$. Sketch the solid $E$ and calculate $\iiint_E y\, dV$.

3. Let $E$ be the solid region $x^2 + y^2 + z^2 \leq 16$. Calculate $\iiint_E z^4\, dV$.

4. Use spherical coordinates to calculate the triple integral $\iiint_E z\, dV$, where $E$ is the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$.