Math 2415

Problem Section #12

Do the problems from 15.9 and 16.1 below.
If you have extra time, finish off the problems from last week.

15.9: Change of Variables Theorem

1. Evaluate $\iint_R (x-y)^2 e^{x+y}\, dx\,dy$ where R is the parallelogram bounded by $x+y=1$, $x+y=3$, $x-y=-2$ and $x-y=1$. **Hint:** Use the Change of Variables Theorem with $u=x+y$ and $v=x-y$.

2. (*Skip this one if you understand Q1.*) Use the Change of Variables Theorem to evaluate the integral $\iint_R y\, dA$, where R is the quadrilateral region bounded by the lines $x+2y=2$, $x+2y=4$, $x=0$, and $y=0$. **Hint:** Let $u=x+2y$ and $v=y$.

3. Use the change of variables formula and an appropriate transformation to evaluate $\iint_R x\, dA$, where R is the square with vertices $(0,0)$, $(2,2)$, $(4,0)$, and $(2,-2)$.

4. Calcuate $\iint_R y^2\, dA$, where R is the region bounded by the ellipse $4x^2+25y^2=1$. Hint: Use the change of variables $u=2x$, $v=5y$.

5. Let D be the region in the first quadrant of the xy-plane bounded by the curves $y=\frac{x}{2}$, $y=x$, $xy=4$ and $xy=9$. Calculate $\iint_D x\, dx\,dy$. **Hint:** Use the change of variables $x=ve^u$, $y=ve^{-u}$.

16.1: Vector Fields

1. Sketch the vector field $\mathbf{F}(x,y)=-x\mathbf{j}$

2. Sketch the vector field $\mathbf{F}(x,y)=x\mathbf{i}+(x-y)\mathbf{j}$

3. Let $f(x,y)=y-x^2$. Calculate the gradient vector field $F=\nabla f$ and sketch it.