Math 2415

Problem Section #13

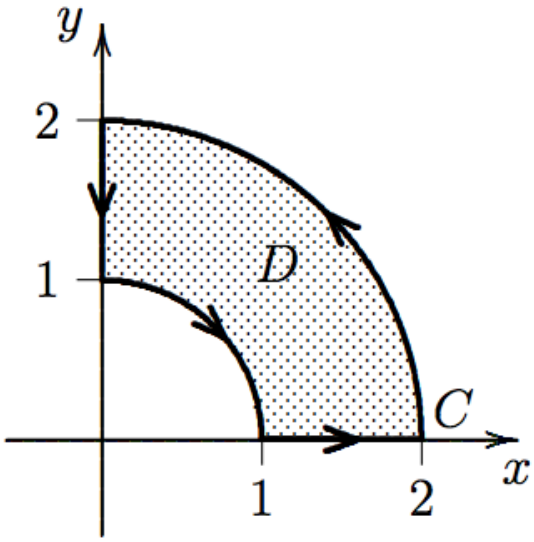
This week we will do problems from 16.2-16.4 as well as review for the Final Exam.

Based on past experience, about 50% of the points on the final exam will be on material from 15.3 onwards. In the next problem session, we will post the same set of exam review problems.

16.2-16.4: Line Integrals, Conservative Vector Fields, Green's Theorem

1. In this problem you will evaluate $\int_C x^2\,ds$, where C is the segment of the curve $y^2 = x^3$ from the origin to $(1,1)$.
   (a) Parametrize the curve C in such a way that x and y are both of the form t^k for some integer k. (You will need a different k for y than for x.)
   (b) This integral is the integral of a function rather than of a vector field. Why?
   (c) Make a rough sketch of the curve C.
   (d) Based on your sketch and the values of the integrand along C, do you expect the integral to be positive, negative, or zero?
   (e) Finally, evaluate the integral.
2. In this problem you will evaluate $\int_C x^2\,dy$, where C is the segment of the curve $x = y^3$ from $(0,0)$ to $(1,1)$.
   (a) Parametrize the curve C.
   (b) This integral is the integral of a vector field rather than of a function. Why?
   (c) What is the formula for the vector field being integrated?
   (d) Make a (rough) sketch of this vector field and add the curve C to your sketch.
   (e) Based on your sketch, do you expect the integral to be positive, negative, or zero?
   (f) Finally, evaluate the integral.
3. Evaluate $\int_C x^2\,dx + y^2\,dy$, where C is the arc of the circle $x^2 + y^2 = 9$ from $(0,3)$ to $(3,0)$ traversed clockwise. This integral represents the work done by a force on a moving particle. What is the formula for the force? Along what path does the particle move?
4. Let $f(x,y) = xe^{x^2+y^2}$. Find $\int_C \nabla f \cdot d\mathbf{r}$, where C is any oriented curve from $(1,1)$ to $(2,2)$.
5. Let $\mathbf{F}_1(x,y) = (2y - x^2e^{-y})\mathbf{i} + 2xe^{-y}\mathbf{j}$ and $\mathbf{F}_2(x,y) = 2xe^{-y}\mathbf{i} + (2y - x^2e^{-y})\mathbf{j}$
   (a) One of these vector fields is conservative. Which one is it and why?
   (b) Find a potential function for the conservative vector field.
   (c) Evaluate $\int\limits_C \mathbf{G} \cdot d\mathbf{r}$ where C is the line segment from $(1,0)$ to $(2,1)$ and $\mathbf{G}$ denotes the conservative vector field you identified in (a).
6. Use Green's theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = (y - \cos y)\,\mathbf{i} + x\sin y\,\mathbf{j}$, and C is the circle $(x-3)^2 + (y+4)^2 = 9$., oriented counter clockwise.

7. Use Green's theorem to evaluate $\int_C xy^2\,dx - x^2y\,dy$ where C is given in the figure.



Final Exam Review

Here are a long list of problems you could work on, many of which are exam questions from past semesters.

Also see Dr. Makhijani's Final Exam Practice Problems, for which there are solutions past exams webpage.

1. Stewart, 15.6.21
2. Stewart, 15.7.21
3. Stewart, 15.7.25 (a)
4. Stewart, 15.8.23
5. Stewart, 15.Review.30
6. Spring 2014 Final Exam # 8
7. Fall 2009 Exam II # 4
8. Fall 2014 Final Exam # 6
9. Spring 2014 Final Exam # 6
10. Spring 2004 Final: 1
11. Spring 2004 Final: 2
12. Spring 2004 Final: 6
13. Spring 2004 Final: 7 (Part d is on 16.6)
14. Spring 2008 Final: 1

15. Spring 2008 Final: 3
16. Spring 2008 Final: 4
17. Spring 2008 Final: 6
18. Spring 2019 Final: 10 (Based on 16.5)
19. Fall 2009 Final: 4 (Based on 16,.6)
20. Fall 2009 Final: 5
21. Fall 2009 Final: 6
22. Fall 2009 Final: 9