Math 2415

Problem Section #7

Make sure you do some problems from each section.

16.6, Parametrized Surfaces

1. **[Parts of Paper Hwk 16.6]:** Let S be the surface with parametrization

$$(x, y, z) = \mathbf{r}(u, v) = u\cos v\,\mathbf{i} + u\sin v\,\mathbf{j} + u\,\mathbf{k} \qquad u \geq 0, \quad 0 \leq v \leq 2\pi.$$

   (a) Show that S is a cone. **Hint:** Find an equation of the form $F(x, y, z) = 0$ for this surface by eliminating u and v from the equations for x, y, and z above.

   (b) Find a parametrization of the tangent plane to the cone at the point where $(u, v) = (2, \pi/4)$.

2. (a) Write down the equation of the form $F(x, y, z) = 0$ for the sphere of radius 2, center $(1, 2, 3)$.

   (b) Show that

$$(x, y, z) = \mathbf{r}(\theta, \phi) = (1 + 2\sin\phi\cos\theta, 2 + 2\sin\phi\sin\theta, 3 + 2\cos\phi)$$

   is a parametrization of this sphere. **Hint:** Substitute the formulae for x, y, and z in terms of θ and ϕ into the function F you obtained in (a) and simplify as much as you can. What does this calculation tell you about where each of the points $(x, y, z) = \mathbf{r}(\theta, \phi)$ lie?

14.3-14.5: Partial Derivatives, Tangent Planes, and Chain Rule

1. Let $f(x, y) = xy + x^2 = x(y + x)$ Calculate f_x, f_y, f_{xy} and f_{xx} at the point $\mathbf{x}_0 = (1, 1)$. Discuss the geometric meaning of each of these derivatives, with the aid of schematic diagrams showing slices of f in planes where x or y are constant.

2. Show that the function $u(x, y) = e^{-x}\cos(y)$ satisfies Laplace's equation $u_{xx} + u_{yy} = 0$.

3. Show that the function $u(x, t) = \cos(kx)\sin(akt)$ satisfies the wave equation $u_{tt} = a^2 u_{xx}$.

4. Find a level set equation of the tangent plane to the function $z = f(x, y) = e^x\cos(xy)$ at $(x_0, y_0) = (0, 0)$. Explain why your solution shows that $e^x\cos(xy) \approx x + 1$ near $(0, 0)$.

5. 14.5.7

6. 14.5.15

7. 14.5.35

14.6: Gradients and Directional Derivatives

1. Let $f(x, y) = e^y \cos(x)$ and let $\mathbf{x}_0 = (\pi/3, 0)$.

   (a) Find the gradient of f.

   (b) Evaluate the gradient of f at $\mathbf{x}_0$, i.e., find $\nabla f(\mathbf{x}_0)$.

   (c) Find the directional derivative of f at $\mathbf{x}_0$ in the direction $\theta = 3\pi/4$.

   (d) Find the directional derivative of f at $\mathbf{x}_0$ in the direction of the vector $\mathbf{v} = (3, 4)$.

   (e) Find the maximum rate of change of f at $\mathbf{x}_0$ and the direction in which it occurs.

   (f) In what direction is the minimum rate of change of f at $\mathbf{x}_0$?

   (g) Use the gradient of f to calculate the equation of the tangent line to the level curve $f(x, y) = \frac{1}{2}$.