Math 2415
Problem Section #8

Work through the problems in order.

14.7A: Local Optimization

A guided example

Consider the function $z = f(x, y) = x^2 + 3y^2 - 2xy - 3x$. The critical points of this function are the points in the xy-plane that satisfy the equations

$$0 = \frac{\partial f}{\partial x} = 2x - 2y - 3 \quad (1)$$
$$0 = \frac{\partial f}{\partial y} = 6y - 2x. \quad (2)$$

Equations (1) and (2) are the equations of a pair of lines in the xy-plane. The critical points are the points that lie on **both** of these lines (since both equations need to hold at a critical point). By sketching the two lines you can see that in this example there can only be one critical point of f.

In general the critical point equations

$$0 = \frac{\partial f}{\partial x}(x, y) \quad (3)$$
$$0 = \frac{\partial f}{\partial y}(x, y) \quad (4)$$

are the equations of a **pair of curves** in the xy-plane. The critical points are the points that lie on both curves. Often you can sketch these two curves and use your sketch to determine how many critical points there are and roughly where they are located. Then you can use this geometric information to guide an algebraic calculation to solve Equations (3) and (4) and thereby find the precise location of the critical points. **Use this combined geometric-algebraic technique in the problems below.**

Problems

1. Find the local maxima, minima, and saddle points of the following functions

   (a) $f(x, y) = x^4 + y^2 + 2xy$
   (b) $f(x, y) = y^3 - 3y + 3x^2y$
   (c) $f(x, y) = x^4 - 2x^2 + y^3 - 6y$
   (d) $f(x, y) = xy + 2x + 2y - x^2 - y^2$
   (e) **Challenge:** $f(x, y) = 2xye^{x^2-y^2}$

14.7B: Global Max/Min

1. Let D be the closed triangle with vertices $(0, 3)$, $(-3, 0)$, and $(3, 0)$. Find the absolute maximum and minimum of the function $f(x, y) = x^2 + y^2 - 2y$ on D.

2. Let D be the rectangle $[0, 4] \times [0, 2]$. Find the absolute maximum and minimum of the function $f(x, y) = 2x^2 + y^2 - 4x - 2y$ on D.

3. Let D be the domain in the xy-plane given by $x \geq 0$, $y \geq 0$, and $x^2 + y^2 \leq 9$. Sketch D. Find the absolute maximum and minimum of the function $f(x, y) = x^2y$ on D.