

Math 2415

Paper Homework #0

This homework set is a review of the parts of pre-calculus and calculus I that we need for MATH 2415. You may use any resources you wish. **Show all work!** Write your solutions on another sheet of paper. Make sure you understand the **theory** behind these problems, not just how to “get the answer”. Before doing Questions 11 and 12 make sure you read the [Surface Sketching Primer](#) (It is slow to load).

1. Let $f(x) = x^2$.
 - (a) Compute $f'(3)$
 - (b) What is the limit definition of $f'(3)$?
 - (c) What does $f'(3)$ mean geometrically?
2. Find
 - (a) $\int_{1/2}^2 \frac{1}{x} dx$
 - (b) $\int_1^2 (x^3 - 6)^4 x^2 dx$ [Integration by u -substitution]
 - (c) $\int_0^2 x^2 e^{3x} dx$ [Integration by parts]
 - (d) $\int_0^\infty x e^{-x^2} dx$
3. On what intervals is $f(x) = 2x^3 + 3x^2 - 12x$ increasing? Also, find the absolute maximum of f on the interval $0 \leq x \leq 3$.
4. State two versions of the Fundamental Theorem of Calculus.
5. Use two triangles and a circle to work out: $\cos \pi/6$, $\sin \pi$, $\tan \pi/4$, $\cot \pi/3$. Hint: See this [Khan Academy video](#)
6. Compute the equation of the tangent line to $y = \cos(x^2)$ at $x = \pi/3$.
7. Graph (a) $2x + 5y = 1$, (b) $y = x^2 - 2x + 3$, (c) $y^2 - 9x^2 = 4$, (d) $16y^2 + 9x^2 = 1$. *Do not use a table of values. Instead use graphing techniques you learned in your pre-calculus course. Need to brush up? See these Khan Academy videos: [Quadratics](#), [Ellipses](#), and [Hyperbolas #1](#) and [Hyperbolas #2](#).*
8. The **Chain Rule** states that if $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$. This means we evaluate f' at the point $g(x)$, g' at the point x and then take the product of the resulting numbers. Use this way of thinking about the chain rule to solve the following problems:
 - (a) Find the derivative of $h(x) = \cos^3(x)$
 - (b) Find the derivative of $h(x) = \sin(x^3)$
 - (c) Let $h(x) = \sin(g(x))$ where $g(0) = 2$ and $g'(0) = 3$. What is $h'(0)$?
 - (d) Suppose that $h(x) = f(g(x))$ and that $g(0) = 2$, $f(0) = 4$, $g'(0) = 3$, $f'(0) = 4$, $g(2) = 6$, $f(2) = -3$, $g'(2) = -5$, $f'(2) = 9$. Find $h'(0)$.

9. Find the area bounded by the curves $y = x/2$ and $y = \sqrt{x}$.
10. In this problem, we take a curve in the yz -plane and rotate it about the z -axis to obtain a surface. Sketch the resulting surfaces in the case that the curve is (a) The circle centered at the origin radius 1, (b) The circle centered at $(y, z) = (2, 0)$ radius 1, (c) The line segment joining $(1, 2)$ to $(3, 4)$.
11. Use the techniques described in the Surface Sketching Primer to sketch
- (a) A rectangle in the horizontal plane, $z = 2$
 - (b) A rectangle in the vertical plane, $y = 1$
 - (c) A parallelepiped
 - (d) A cereal bowl with a circular rim
 - (e) A cylinder aligned with the y -axis
 - (f) An icecream cone with a spherically shaped scoop of icecream in the top