

Math 2415
Paper Homework #12

1. 15.7, Triple Integrals in Cylindrical Coordinates

- (a) Let E be the solid region in the first octant (*i.e.*, where $x \geq 0$, $y \geq 0$, $z \geq 0$) that is inside the cylinder $x^2 + y^2 = 1$ and below the plane $x + z = 1$. Calculate $\iiint_E y \, dV$. **Hint:** This problem was in PS #12.
- (b) Find $\iiint_E \sqrt{x^2 + y^2} \, dV$ where E is the solid region above the xy -plane, below the paraboloid $z = 8 - x^2 - y^2$ and *outside* the cylinder $x^2 + y^2 = 1$. Sketch the solid E .

2. 15.8, Triple Integrals in Spherical Coordinates

- (a) Find $\iiint z \, dV$ where E is the solid region that is inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$. **Hint:** This problem was in PS #12.
- (b) Find the volume of the region inside the ball $x^2 + y^2 + z^2 \leq R^2$ that lies between the planes $y = 0$ and $y = \sqrt{3}x$ in the first octant.

3. 15.9, Change of Variables Theorem:

- (a) Evaluate $\iint_R (x - y)^2 e^{x+y} \, dx \, dy$ where R is the parallelogram bounded by $x + y = 1$, $x + y = 3$, $x - y = -2$ and $x - y = 1$. **Hint:** This problem was in PS #12.
- (b) Use the change of variables $u = y/x^2$, $v = x/y^2$ to find the area of the region in the first quadrant that is bounded by the curves $y = x^2$, $y = 3x^2$, $x = y^2$ and $x = 4y^2$.