Math 2415

Paper Homework #14

1. **16.3, Conservative Vectors and FTC for Line Integrals:** Consider the vector field

$$\mathbf{F}(x, y) \;=\; (1 - ye^{-x})\mathbf{i} + e^{-x}\mathbf{j}.$$

   (a) Show that **F** is a conservative vector field.

   (b) Find a function f so that $\mathbf{F} = \nabla f$.

   (c) Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $\mathbf{r}(t) = e^t\mathbf{i} + \sin t\mathbf{j}$, for $0 \le t \le \pi/2$.

2. **16.4, Green's Theorem:** Use Green's Theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y) = (6y + x)\mathbf{i} + (y + 2x)\mathbf{j}$ and C is the circle $(x - 2)^2 + (y - 3)^2 = 4$ traversed clockwise.

3. **16.5, Curl and Divergence:** Compute the divergence and curl of the following vector fields

   (a) $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$.

   (b) $\mathbf{F} = \frac{x\mathbf{i}+y\mathbf{j}+z\mathbf{k}}{(x^2+y^2+z^2)^{3/2}}$

4. In this multiple choice problem provide a justification for your answer.

   Let $\vec{F}(x, y, z)$ be a vector field and let $f(x, y, z)$ be a scalar function. If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, which of the following is not defined?

   (a) $\nabla \times f$

   (b) $\nabla \times \vec{F} + \nabla f$

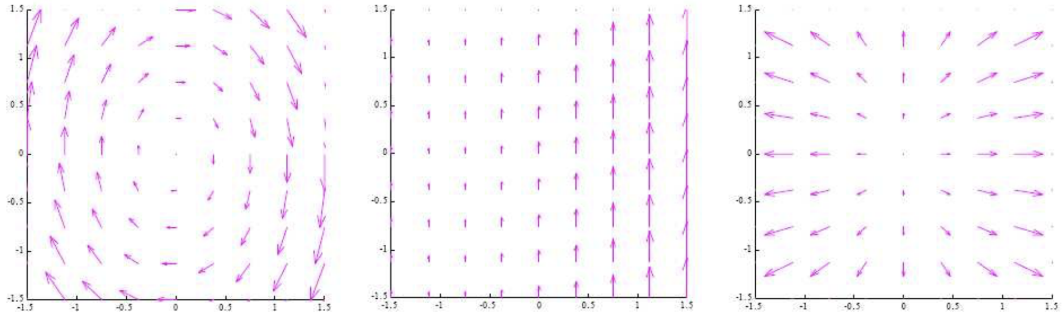
   (c) $\nabla \times (\vec{r} \times \nabla f)$

   (d) $f + \nabla \cdot \vec{F}$

   (e) More than one of the above

5. In the multiple choice problem on the next page provide a justification for your answer.

The pictures below show top views of three vector fields, all of which have no z component. Which one has the curl pointing in the positive $\hat{k}$ direction at the origin?



(a) the one on the left

(b) the one in the middle

(c) the one on the right

(d) none of them