Math 2415

Problem Section #12

Do at least some problems from each section.
Recommended (in this order): 15.7 and 15.8 (1,2,4), 15.9.1, 15.9.3, 15.9.4
15.3.2, 15.3.4, 15.6.1a, 15.6.2, 15.6.3

You will have the opportunity to do the remaining problems next week.

15.3, Double Integrals in Polar Coordinates

1. Evaluate $\iint_D e^{x^2+y^2}\, dA$, where D is the region in the 1st quadrant between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

2. Evaluate $\iint_D \cos(x^2+y^2)\, dA$, where D is the region bounded by the semicircle $x = \sqrt{9-y^2}$ and the y-axis.

3. Calculate the volume of the solid under $z = x^2 + y^2$ and above $x^2 + y^2 \leq 16$.

4. Calculate the volume of the solid below the plane $x + 2y + 3z = 6$ and above $x^2 + y^2 \leq 1$.

5. Evaluate the integral by converting to polar coordinates: $\int_0^R \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} (x+2y)\, dy\, dx$.

15.6, Triple Integrals in Rectangular Coordinates

1. Sketch the region bounded by the following surfaces. Each pair of the surfaces intersects in a curve. Be sure to include these curves in your sketch. Then use a triple integral to calculate the volume of the solid.

   (a) $z = x^2 + y^2$, $x = 0$, $y = 0$, $z = 0$, $x + y = 1$.

   (b) $x = z^2$, $x = 8 - z^2$, $y = 1$, $y = 3$.

2. Set up an iterated triple integral to calculate $\iiint_E x^2 dV$ where E is the solid bounded by $y = z^2$, $y = z$, $x + y + z = 2$, $x = 0$.

3. Evaluate $\iiint_E y\, dV$, where E is the solid bounded by the surfaces $z = 2 - x^2$, $z = x^2 - 2$, $y = 0$ and $y = 1$.

4. Find the volume of the solid enclosed by the generalized cylinder $z = x^2$ and the planes $y = 0$ and $y + z = 2$. **Hint:** When sketching this surface it may be helpful to know that the intersection of the slanted plane $y + z = 2$ and the generalized cylinder $z = x^2$ is a parabola in the slanted plane.

15.7 and 15.8, Triple Integrals in Cylindrical and Spherical Coordinates

1. Use cylindrical coordinates to find the volume of the solid that lies outside the cylinder $x^2 + y^2 = 3$ and inside the sphere $x^2 + y^2 + z^2 = 4$.

2. Let E be the solid region in the first octant (*i.e.*, where $x \geq 0$, $y \geq 0$, $z \geq 0$) that is inside the cylinder $x^2 + y^2 = 1$ and below the plane $x + z = 1$. Calculate $\iiint_E y\, dV$.

3. Let E be the solid region $x^2 + y^2 + z^2 \leq 16$. Calculate $\iiint_E z^4 \, dV$.

4. Use spherical coordinates to calculate the triple integral $\iiint_E z \, dV$, where E is the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$.

15.9: Change of Variables Theorem

1. Evaluate $\iint_R (x-y)^2 e^{x+y} \, dx dy$ where R is the parallelogram bounded by $x+y=1$, $x+y=3$, $x-y=-2$ and $x-y=1$. **Hint:** Use the Change of Variables Theorem with $u = x+y$ and $v = x-y$.

2. (*Skip this one if you understand Q1.*) Use the Change of Variables Theorem to evaluate the integral $\iint_R y \, dA$, where R is the quadrilateral region bounded by the lines $x + 2y = 2$, $x + 2y = 4$, $x = 0$, and $y = 0$. **Hint:** Let $u = x + 2y$ and $v = y$.

3. Use the change of variables formula and an appropriate transformation to evaluate $\iint_R x \, dA$, where R is the square with vertices $(0, 0)$, $(2, 2)$, $(4, 0)$, and $(2, -2)$.

4. Calcuate $\iint_R y^2 \, dA$, where R is the region bounded by the ellipse $4x^2 + 25y^2 = 1$. Hint: Use the change of variables $u = 2x$, $v = 5y$.

5. Let D be the region in the first quadrant of the xy-plane bounded by the curves $y = \frac{x}{2}$, $y = x$, $xy = 4$ and $xy = 9$. Calculate $\iint_D x \, dx dy$. **Hint:** Use the change of variables $x = ve^u$, $y = ve^{-u}$.