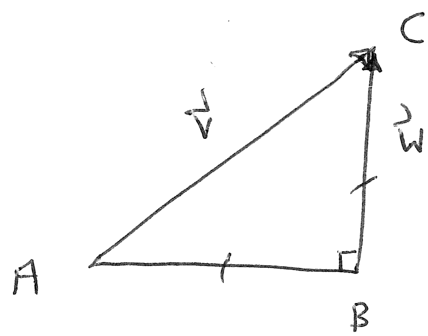
Math 2415
Paper Homework #1

Some of the paper homework problems in this course are "scaffolded". This means that we break a harder problem down into several manageable parts. When you are working on such problems it helps to work out how the answers to the earlier parts help you tackle the later parts. Schematic diagrams may help you solve each part and also connect different parts of the problem. A schematic diagram shows the relationships between the different nouns in the problem statement, but not in a completely realistic way.

1. (a) Describe and sketch the curve in $\mathbb{R}^2$ represented by $(x-1)^2 + (y+3)^2 = 4$.
   (b) Describe and sketch the surface in $\mathbb{R}^3$ represented by $(x-1)^2 + (y+3)^2 = 4$.

2. Suppose you have a parallelogram with vertices P, Q, R, and S so that the displacement vector from P to Q equals the displacement vector from S to R. If $P = (2,0,1)$, $Q = (3,1,0)$ and $R = (4,3,5)$ find the coordinates of the point, S. Draw a schematic diagram showing how you arrived at your answer.

3. The triangle ABC in the figure below is an isoceles triangle for which the length of the hypotenuse is 1. Let $\mathbf{v} = \overrightarrow{AC}$ and $\mathbf{w} = \overrightarrow{BC}$. Calculate (a) $\mathbf{v} \cdot \mathbf{w}$, (b) the scalar projection of $\mathbf{w}$ onto $\mathbf{v}$, and (c) the vector projection of $\mathbf{v}$ onto $\mathbf{w}$.



4. Suppose that AB is the diameter of a circle with center O and that C is a point on the circle, with C not equal to either A or B. (See figure below.) Use the following steps to prove that the vectors $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are orthogonal.

   (a) Why can you assume that the point O is the origin and that the line segment $\overline{AB}$ is on the x-axis?
   (b) Suppose that angle $\angle BOC = \theta$ and that the radius of the circle is R. Find a formula for the vector $\mathbf{c} = \overrightarrow{OC}$ in terms of R and θ.

(c) Let $\mathbf{a} = \overrightarrow{AC}$ and $\mathbf{b} = \overrightarrow{BC}$. Find formulae for the vectors $\mathbf{a}$ and $\mathbf{b}$ in terms of R and θ.

(d) Calculate the dot product of $\mathbf{a}$ and $\mathbf{b}$. What can you conclude?