Math 2415

Paper Homework #13

1. Sketch the following vector fields

   (a) $\mathbf{F}(x, y) = (2y - x)\mathbf{i} + (x + y)\mathbf{j}$

   (b) $\mathbf{F}(x, y) = \nabla f$ where $f(x, y) = 4x^2 + y^2$

   Hint for (a): To sketch the vector field $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$, find the region in the (x, y)-plane where $P > 0$ and $Q > 0$. In this region $\mathbf{F}$ points roughly north-east. Repeat for the regions where (i) $P > 0$ and $Q < 0$, (ii) $P < 0$ and $Q > 0$, and (iii) $P < 0$ and $Q < 0$.

2. Evaluate $\int_C f\, ds$ where C has parametrization $\mathbf{r}$ with $\mathbf{r}(t) = (t^3, t^2, t)$ for $0 \leq t \leq 1$ and $f(x, y, z) = 2z + 9yz$.

3. Evaluate $\int_C yz\, dy + xy\, dz$ where C is the curve given by $x = \sqrt{t}$, $y = t$, $z = t^2$ for $1 \leq t \leq 2$.

4. Evaluate $\int_C [(2x + 3y)\mathbf{i} + (3x + 2y)\mathbf{j} + 3y^2\mathbf{k}] \cdot d\mathbf{r}$ where C is the line segment from $(2, -1, 3)$ to $(4, 2, -1)$.

5. Let C be the curve of intersection of the plane $2x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$.