Math 2415
Problem Section #13

This week we will do problems from 16.1-16.2 as well as review for the Final Exam.

Based on past experience, about 50% of the points on the final exam will be on material from 15.3 onwards. In the next problem session, we will post the same set of exam review problems.

16.1: Vector Fields

1. Sketch the vector field $\mathbf{F}(x, y) = -x\mathbf{j}$.
2. Sketch the vector field $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$, where $P(x, y) = x$ and $Q(x, y) = x - y$. Use the following steps
   - Find the region in the (x, y)-plane where $P > 0$ and $Q > 0$. In this region **F** points roughly north-east. Draw in some horizontal lines in this region and plot the vectors at several points along this line. How do the direction and magnitude of the vectors change as you move along each horizontal line?
   - Repeat for the region where $P > 0$ and $Q < 0$.
   - Now you know what the vector field looks like everywhere in the right half of the plane. Use algebra to show that the vector field is odd in that $\mathbf{F}(-x, -y) = -\mathbf{F}(x, y)$. Use this symmetry to work out what the vector field looks like in the left half plane. For example, $\mathbf{F}(-2, -3) = -\mathbf{F}(2, 3)$.
3. Let $f(x, y) = y - x^2$. Calculate the gradient vector field $F = \nabla f$ and sketch it. **Hint:** If you know the vectors on the parabola $y = x^2$ you can easily work out what they are everywhere.

16.2: Line Integrals

1. In this problem you will evaluate $\int_C x\, ds$, where C is the segment of the curve $y^2 = x^3$ from the origin to $(1, -1)$.
   (a) Make a rough sketch of the curve C.
   (b) Parametrize the curve C in such a way that x and y are both given in terms of t^k for some integer k. (You will need a different k for y than for x.)
   (c) This integral is the integral of a function rather than of a vector field. Why?
   (d) Based on your sketch and the values of the integrand along C, do you expect the integral to be positive, negative, or zero?
   (e) Finally, evaluate the integral.
   (f) **Optional Challenge:** Repeat for $\int_C y\, ds$. In this case you will need to do a trigonometric substitution. Yikes!
2. In this problem you will evaluate $\int_C x^2\, dy$, where C is the segment of the curve $x = y^3$ from $(0, 0)$ to $(1, 1)$.
   (a) Parametrize the curve C.

(b) This integral is the integral of a vector field rather than of a function. Why?

(c) What is the formula for the vector field being integrated?

(d) Make a (rough) sketch of this vector field and add the curve C to your sketch.

(e) Based on your sketch, do you expect the integral to be positive, negative, or zero?

(f) Finally, evaluate the integral.

3. Evaluate $\int_C y^2\,dx + x^2\,dy$, where C is the arc of the circle $x^2 + y^2 = 9$ from $(0, 3)$ to $(3, 0)$ traversed clockwise. This integral represents the work done by a force on a moving particle. What is the formula for the force? Along what path does the particle move?

Final Exam Review

Here are a long list of problems you could work on, many of which are exam questions from past semesters.

Also see Dr. Makhijani's Final Exam Practice Problems, for which there are solutions past exams webpage.

1. Stewart, 15.6.21
2. Stewart, 15.7.21
3. Stewart, 15.7.25 (a)
4. Stewart, 15.8.23
5. Stewart, 15.Review.30
6. Spring 2014 Final Exam # 8
7. Fall 2009 Exam II # 4
8. Fall 2014 Final Exam # 6
9. Spring 2014 Final Exam # 6
10. Spring 2004 Final: 1
11. Spring 2004 Final: 2
12. Spring 2004 Final: 6
13. Spring 2004 Final: 7 (Part d is on 16.6)
14. Spring 2008 Final: 1
15. Spring 2008 Final: 3
16. Spring 2008 Final: 4
17. Spring 2008 Final: 6
18. Spring 2019 Final: 10 (Based on 16.5)
19. Fall 2009 Final: 4 (Based on 16,.6)

20. Fall 2009 Final: 5
21. Fall 2009 Final: 6
22. Fall 2009 Final: 9