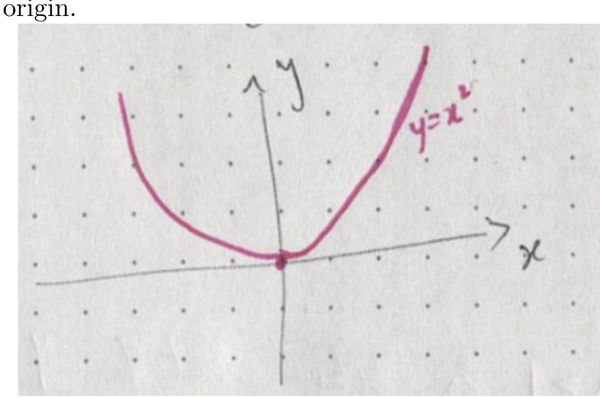
How to draw conics

March 9, 2023

These directives are only for sketching parabolas, ellipses, and hyperbolas schematically. All constants below are positive.

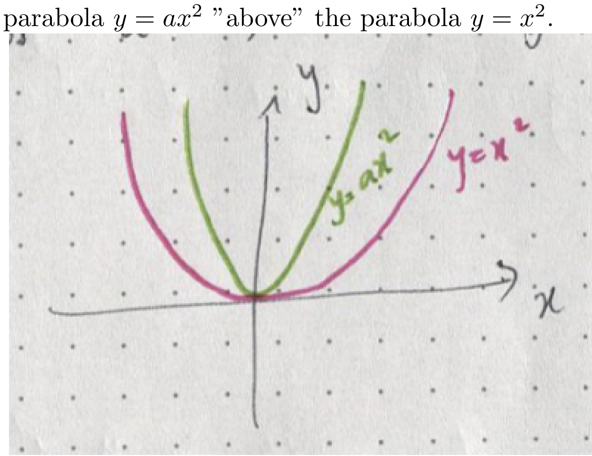
1 Parabolas

We shall assume that you know how to draw $y = x^2$ in the $x - y$ plane. Note that all the parabolas below pass through the origin.

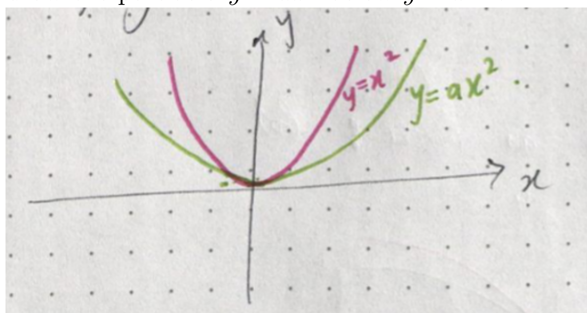


1.1 $y = ax^2$

For $a > 1$, you draw the parabola $y = ax^2$ "above" the parabola $y = x^2$.



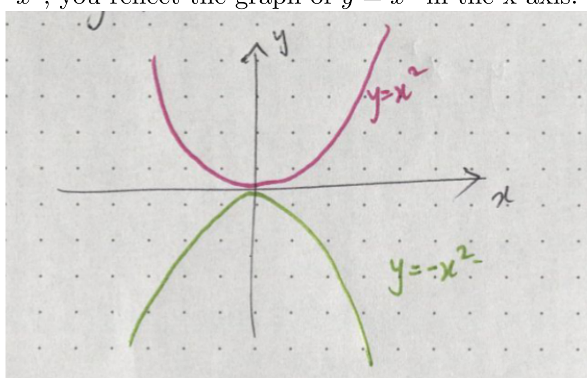
When $1 > a > 0$, you draw the parabola $y = ax^2$ below $y = x^2$ it.



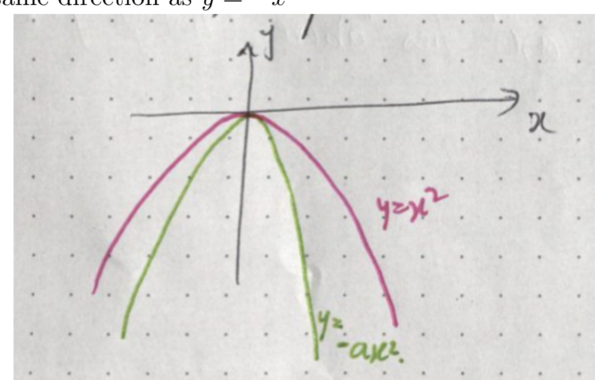
For $b > a > 1$, you draw the parabola $y = bx^2$ above $y = ax^2$.

1.2 $y = -ax^2$

To get the graph of $y = -x^2$, you reflect the graph of $y = x^2$ in the x-axis.

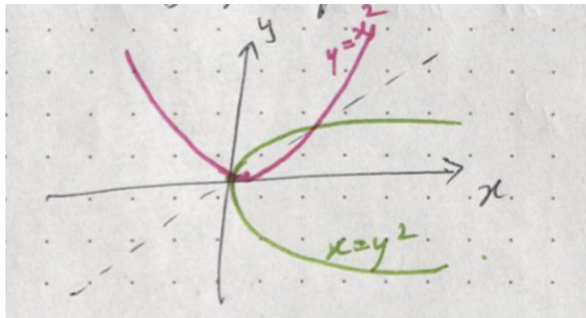


The graph of $y = -ax^2$ for $a > 1$ is below that of $y = -x^2$. For $0 < a < 1$, it is above the graph of $y = x^2$ (opening in the same direction as $y = -x^2$



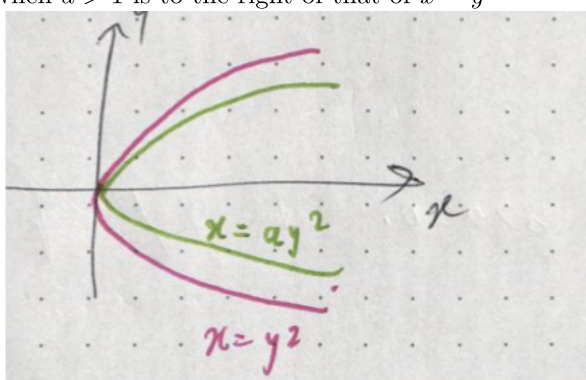
1.3 $x = y^2$

The graph of $x = y^2$ is obtained by reflecting the graph of $y = x^2$ in the line $y = x$. It may help to think of this as rotating the graph of $y = x^2$ clockwise by 90° and relabelling the axes so that y is up and x is to the right.

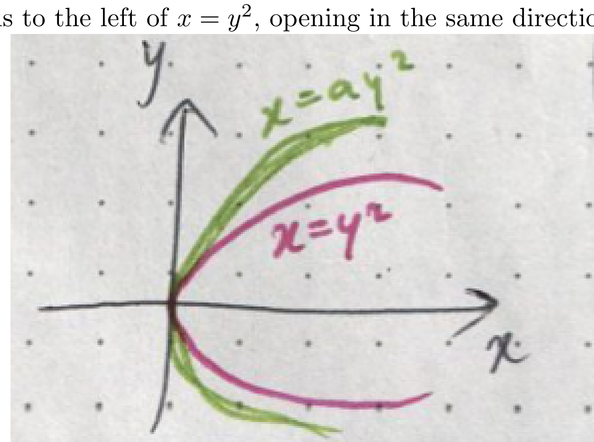


1.4 $x = ay^2$

The graphs for $y = ax^2$ when $a > 1$ is to the right of that of $x = y^2$

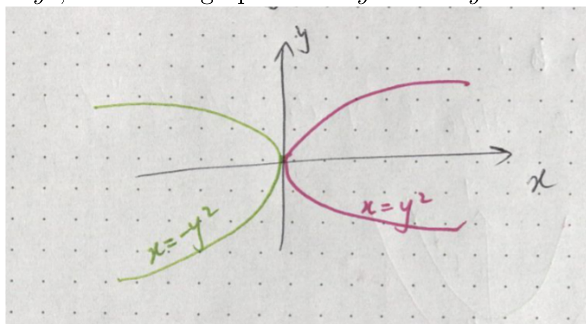


and for $1 > a > 0$ it is to the left of $x = y^2$, opening in the same direction as $x = y^2$.

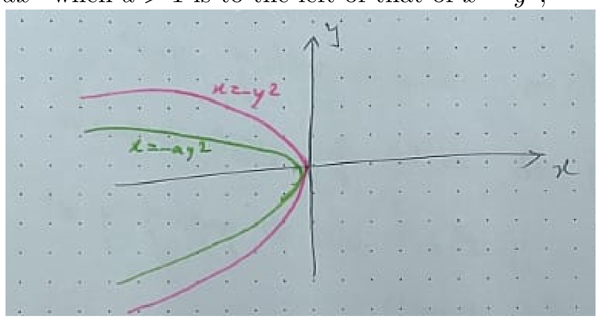


1.5 $x = -ay^2$

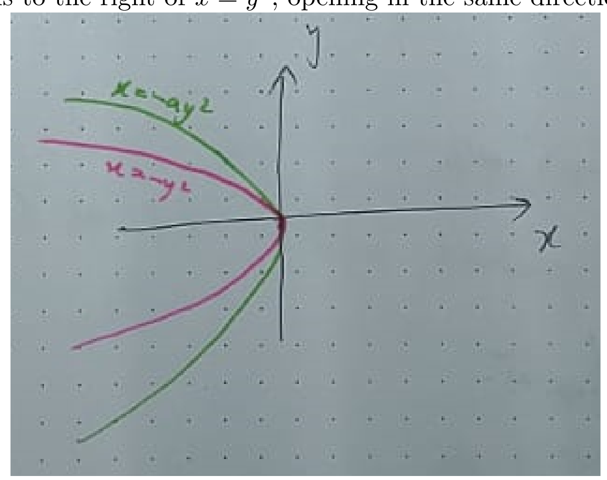
To sketch the graph of $x = y^2$, reflect the graph of $x = y^2$ in the $y-$axis.



The graphs for $y = -ax^2$ when $a > 1$ is to the left of that of $x = y^2$,



and for $1 > a > 0$ it is to the right of $x = y^2$, opening in the same direction as $x = y^2$.

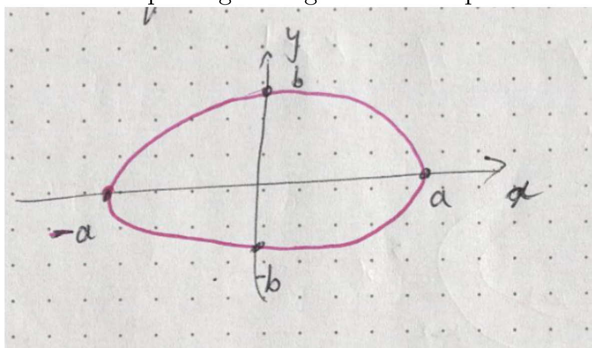


2 Ellipses

2.1 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

To draw $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we first find the points where the curve intersects the axes: these points are $(a, 0), (-a, 0), (0, b), (0, -b)$.

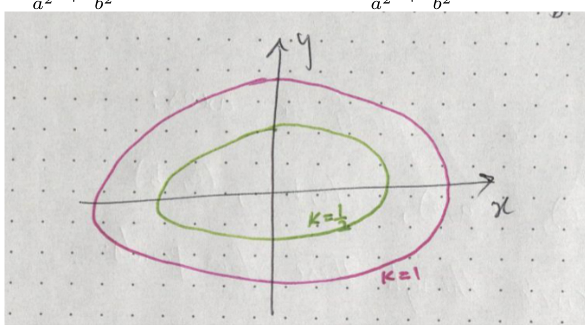
We then draw a "squished" circle passing through these four points.



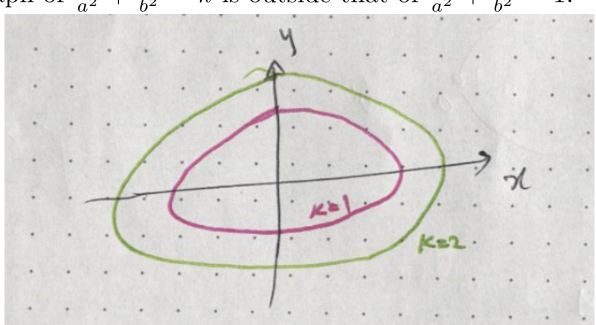
2.2 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = k$

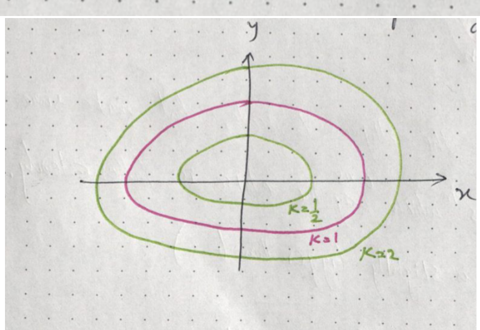
There is nothing to do when $k < 1$. For $k = 0$, we just get the point $(0, 0)$.

For $k < 1$, the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = k$ is inside that of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

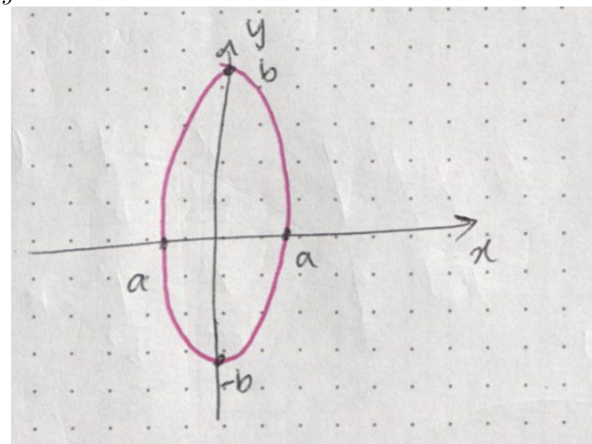


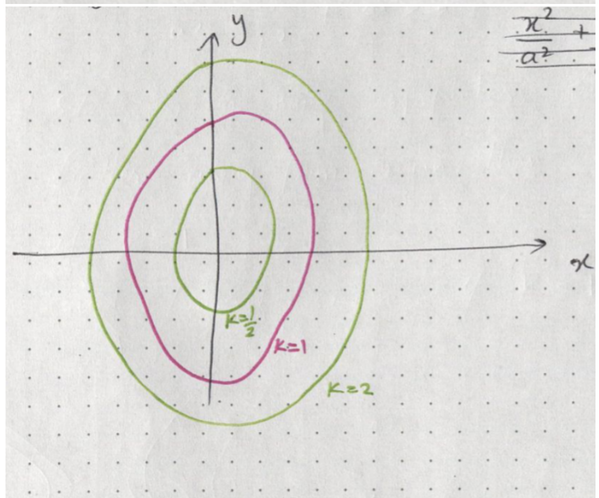
and for $k > 1$, the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = k$ is outside that of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.





The figures above were drawn for $a > b$, when $a < b$, the same trend holds, but this time, the ellipses are fatter in the $y-$direction.





2.3 Remark

When drawing ellipses, your sketch does not have to be to scale but make sure that it shows which direction the ellipse is thinner/fatter clearly. You should do this even if a is ever so slightly larger/smaller than b. Also note that when a=b, we have a circle.

3 Hyperbolas

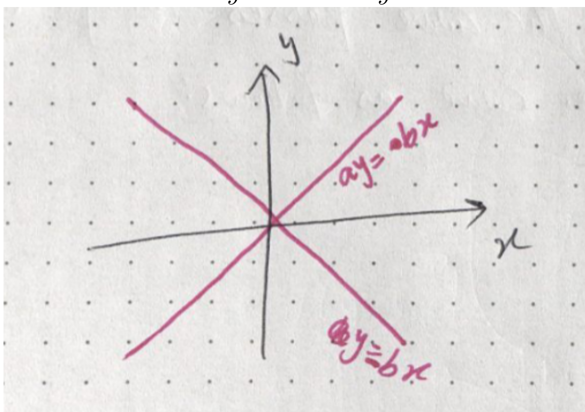
3.1 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

To sketch $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, proceed as follows:

- Set the constant term in the equation to zero:

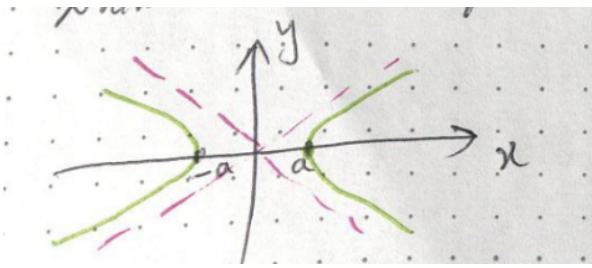
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

  to find the asymptotes of the curve: $ay = bx$ and $ay = -bx$.



- Find which axis the curve intersects. Here setting $y = 0$ gives $x = a$ and $x = -a$, so the curve intersects the $x-$axis.

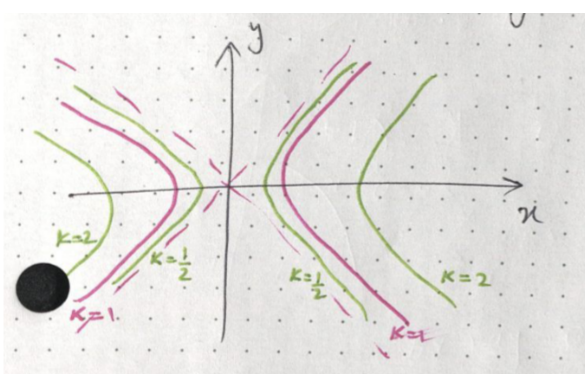
- Draw the two branches of the hyperbola passing through $(a, 0)$ and $(-a, 0)$:



3.2 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = k$

Here, the hyperbola as the same asymptotes as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

For $k > 1$, the curve is further away from the origin than $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and for $k < 1$ it is more towards the origin.



3.3 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

To sketch $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$:

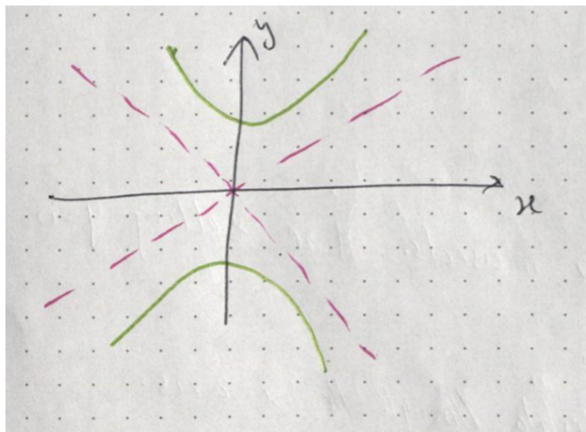
- Set the constant term in the equation to zero:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

  to find the asymptotes of the curve: $ay = bx$ and $ay = -bx$.

- Find which axis the curve intersects. Here setting $x = 0$ gives $y = b$ and $y = -b$, so the curve intersects the $y-$axis.

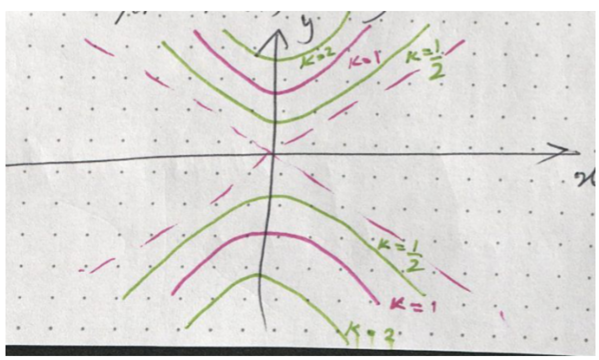
- Draw the two branches of the hyperbola passing through $(0, b)$ and $(0, -b)$:



3.4 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -k$

Here, the hyperbola as the same asymptotes as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

For $k > 1$, the curve is further away from the origin than $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ and for $k < 1$ it is more towards the origin.



3.5 $xy = 1$

To sketch this hyperbola, as before

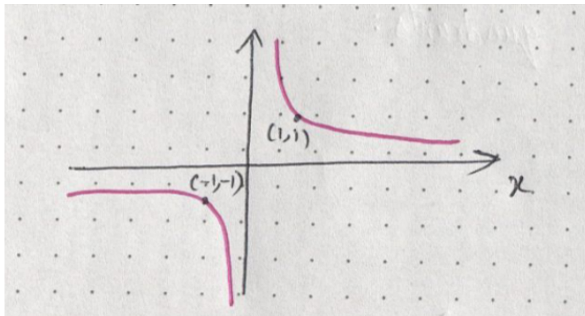
- Set the constant term in the equation to zero:

$$xy = 0$$

  to find the asymptotes $x = 0$ and $y = 0$.

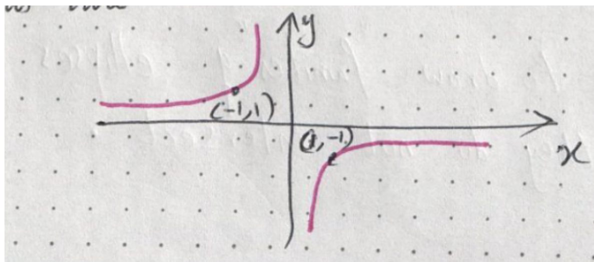
- Find a point that lies on the hyperbola. (1,1) works, so does (-1,-1)

- Draw the hyperbola as below:



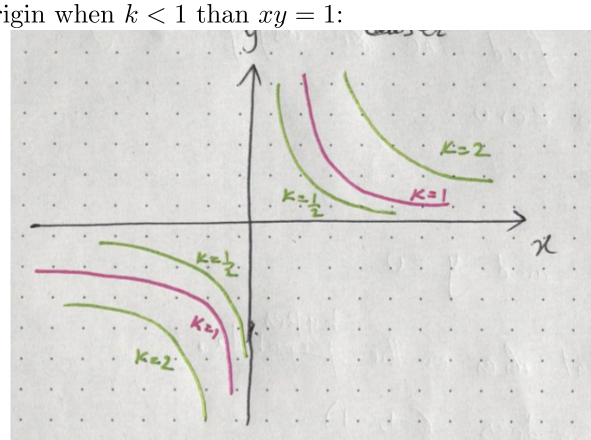
3.6 $xy = -1$

This has the same asymptotes as $xy = 1$ (namely, $x = 0$ and $y = 0$) but now the points $(1, -1)$ and $(-1, 1)$ lie on the hyperbola instead. Draw this as follows:



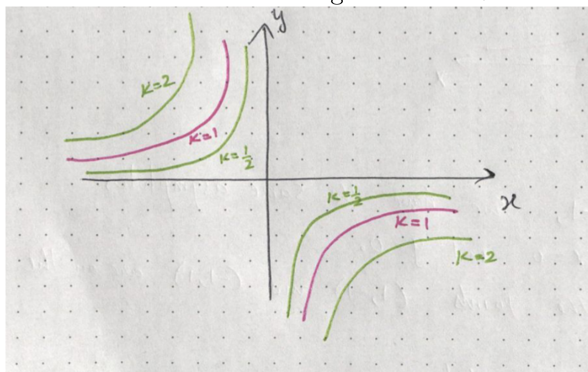
3.7 xy=k

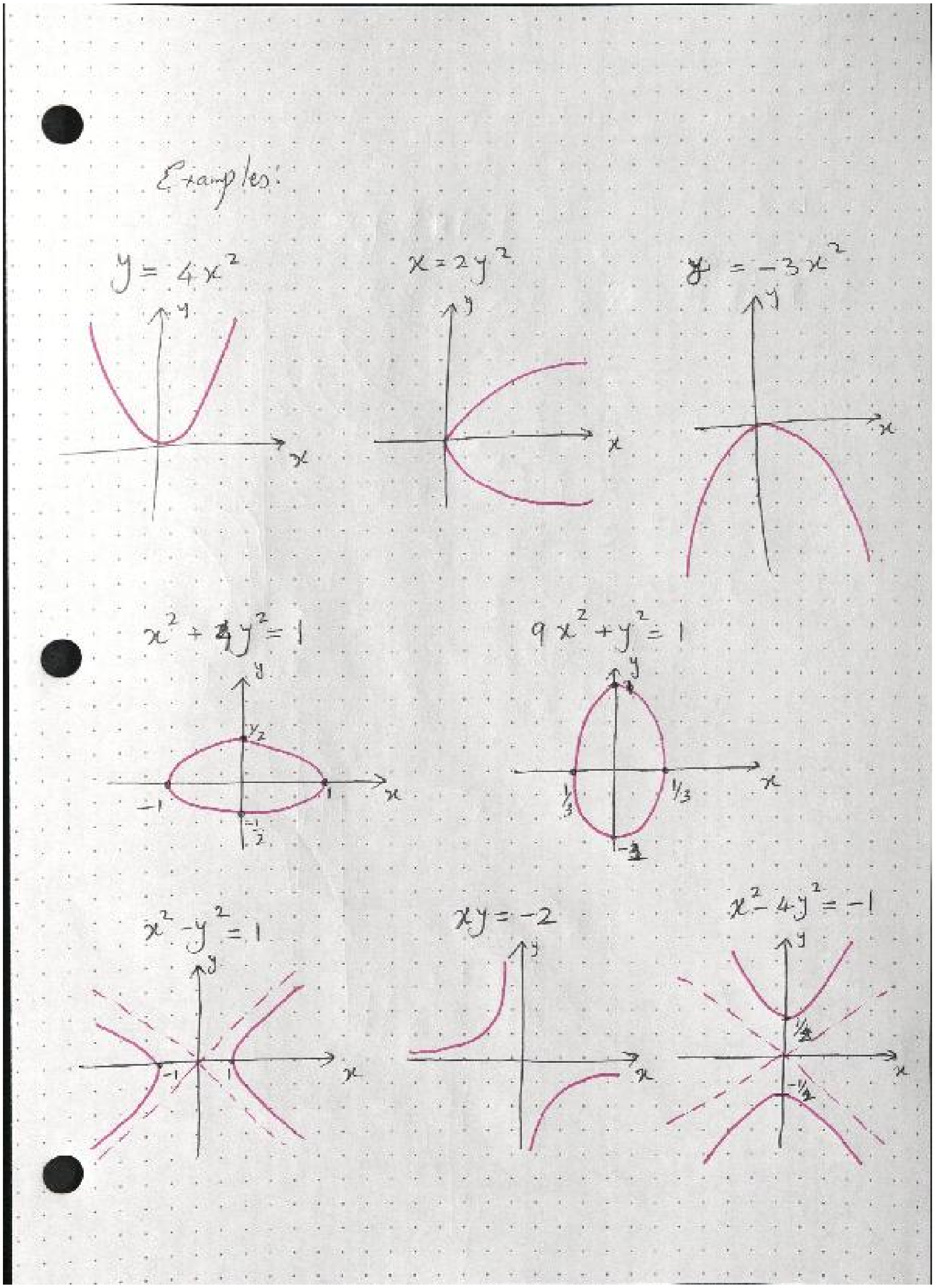
This, again, has the same asymptotes as $xy = 1$. The curve is further away from the origin when $k > 1$ and more towards the origin when $k < 1$ than $xy = 1$:



3.8 xy=-k

This has the same asymptotes as $xy = -1$ (which are the same for $xy = 1$). The curve is further away from the origin when $k > 1$ and more towards the origin when $k < 1$ than $xy = 1$.




Examples:
$y = 4x^2$
$x = 2y^2$
$y = -3x^2$
$x^2 + 4y^2 = 1$
$9x^2 + y^2 = 1$
$x^2 - y^2 = 1$
$xy = -2$
$x^2 - 4y^2 = -1$