

MATH 2415

Learning Outcomes

1. Vectors

- (a) Define concepts of point and vector and explain differences and similarities between them.
- (b) Recognize when it is appropriate to use a point and when to use a vector in problem solving.
- (c) Memorize formulae for length and direction of vector

2. Dot and Cross Product (Vector Algebra)

- (a) Memorize algebraic definitions and explain geometric meanings of dot and cross products
- (b) Compute dot and cross products given either algebraic or geometric information.
- (c) Apply dot or cross product to determine angles between vectors, orientation of axes, areas of triangles and parallelograms in space, scalar and vector projections, and volumes of parallelepipeds. Reproduce sketches and written explanations deriving these formulae.

3. Lines and Planes

- (a) Specify different sets of data required to specify a line or a plane.
- (b) Memorize formulae for parametric equation of a line in space and explain geometrical and physical interpretations.
- (c) Memorize formulae for parametric, level set, and graph-of-function descriptions of plane in space and provide geometrical interpretations with the aid of sketches. Convert between these algebraic representations of a plane and recognize which to apply in problem solving contexts.
- (d) Sketch specific lines and planes described using algebraic formulae
- (e) Solve problems involving geometric relationships between lines and/or planes.

4. Cylindrical and spherical coordinates

- (a) Memorize change of coordinate formulae between rectangular and cylindrical coordinate systems.
- (b) Memorize change of coordinate formulae between rectangular and spherical coordinate systems.
- (c) Derive change of coordinate formulae with the aid of schematic diagrams
- (d) Identify coordinate surfaces in cylindrical and spherical coordinate systems
- (e) Convert equations between rectangular, cylindrical and spherical coordinate systems.

5. Quadric Surfaces

- (a) Recognize when level set and graph-of-function formulae are quadric surfaces
- (b) Classify quadric surfaces as generalized cylinders, surfaces of revolution or general quadric surfaces using algebraic or geometrical representations.
- (c) Sketch quadric surfaces that are generalized cylinders
- (d) Sketch quadric surfaces that are surfaces of revolution using cylindrical coordinates.
- (e) Recognize and plot families of quadric curves in the plane
- (f) Construct geometric representations of quadric surfaces from slices/traces
- (g) Recognize symmetry properties of quadric surfaces, including rotational, scaling, and axes-swapping symmetries and apply in problem solving.

6. Curve Sketching

- (a) Understand concept of parametrized curve from algebraic, geometric and physical standpoints
- (b) Match algebraic formulae and geometric representations of curves
- (c) Sketch circles, ellipses and graphs of functions starting from algebraic formulae in the form of a parametrized curve
- (d) Sketch simple curves in space, such as a helix
- (e) In simple cases, work out parametrizations of curves that are given as the intersection of two surfaces.

7. Derivatives of Parametrized Curves

- (a) Define derivative of a parametrized curve and illustrate geometric (tangent line) and physical meanings (velocity vector) with the aid of sketches
- (b) Memorize formula for arclength and compute in simple cases
- (c) Understand why it is generally hard to compute arclength and explain what to do about that.
- (d) Perform calculations involving derivatives of curves and vector algebra in the context of problem solving.

8. Visualization of functions of several variables

- (a) Sketch graphs of functions of two variables or level sets of functions of three variables. (Simple cases only.)
- (b) Sketch level curves of the graph of a function of two variables
- (c) Match algebraic formulae, sketches of graphs, and sketches of level curves

9. Parameterized surfaces

- (a) Understand concept of a parametrized surface and its grid curves (mesh)
- (b) Convert parametric equations for a surface to level set equations
- (c) Use cylindrical and spherical coordinate systems to derive parametric equations for surfaces
- (d) Compute parameterizations of the tangent plane at a point to a parametrized surface

10. Partial Derivatives

- (a) Memorize definition of partial derivative and illustrate geometric meaning with the aid of sketches
- (b) Calculate first and second partial derivatives
- (c) Provide geometrical meaning of second partial derivative with respect to x and of mixed second partial derivatives.
- (d) Memorize Theorem on equality of mixed second partial derivatives
- (e) Understand notion of tangent plane as “zoomed-in” linear approximation to surface
- (f) Memorize equation for tangent plane to graph of function and compute it.
- (g) Memorize and apply the version of the chain rule for functions on curves that is given in terms of the gradient.
- (h) Use tree diagrams to formulate versions of the multivariable chain rule and apply to compute derivatives of compositions of functions of several variables

11. Gradient and Directional Derivative

- (a) Memorize definition of directional derivative and gradient and illustrate geometric meanings with the aid of sketches
- (b) Memorize theorem relating directional derivative to gradient and reproduce proof
- (c) Calculate directional derivatives and gradients
- (d) Apply gradient to solve problems involving steepest ascent and normal vectors to level curves

12. Optimization

- (a) Explain without use of derivatives what it means for a function of two variables to have a local maximum/minimum
- (b) Define notion of critical point and explain relationship to local max/min
- (c) Identify number and approximate locations of critical points by finding intersections of curves in the plane on which partial derivatives are zero.
- (d) Compute critical points algebraically

- (e) Memorize second derivative test and apply in problem solving
- (f) State and apply method for finding absolute maxima and minima of a function of two variables on a closed bounded domain.
- (g) Develop a conceptual understanding of what it means to optimize a function of two variables subject to a single constraint
- (h) Apply the method of Lagrange Multipliers to solve such constrained optimization problems
- (i) Solve Lagrange Multipliers problems qualitatively using geometric arguments and sketches.

13. Double and Triple Integrals

- (a) Compute double integrals over rectangles and “type I and II” regions in the plane
- (b) Compute double integrals over a sector of an annulus using polar coordinates
- (c) Memorize the statement of the change of variables theorem for double integrals, illustrate its geometric meaning with the aid of sketches, and apply it to compute integrals over regions that are neither type I nor type II.
- (d) Memorize the formulae for integration in cylindrical and spherical coordinates.
- (e) Sketch solids bounded by a collection of surfaces by identifying and sketching the curves given by pairwise intersections of these surfaces.
- (f) Identify the most appropriate coordinate system in which to compute a triple integral.
- (g) Compute triple integrals in rectangular, cylindrical and spherical coordinates.

14. Vector Calculus

- (a) Explain the concept of a vector field and make sketches of simple vector fields in the plane
- (b) Memorize formulae for integrals of functions and vector fields over parametrized curves and compute such integrals
- (c) Provide geometric and physical explanations of the integral of a function over a curve.
- (d) Provide geometric and physical explanations of the integral of a vector field over a curve.
- (e) Memorize statement and understand proof of Fundamental Theorem of Calculus for functions on curves.
- (f) Explain concept of a conservative vector field, state and apply theorems that give necessary and sufficient conditions for when a vector field is conservative, and describe applications to physics
- (g) Memorize Green’s Theorem, and make sketch illustrating it. Explain how Green’s Theorem is a generalization of the Fundamental Theorem of Calculus.

- (h) Memorize definitions of curl and divergence of vector field and compute them
- (i) Explain physical meaning of curl and divergence in terms of fluid flow.
- (j) Understand concept of a parametrized surface and special example of the longitude-latitude parametrization of a sphere.
- (k) Understand concept of the integrals of functions and vector fields over parametrized surfaces and compute them in simple examples
- (l) Recognize the statements of Stokes' Theorem and the Divergence Theorem and understand how they are generalizations of the Fundamental Theorem of Calculus. Be aware of applications of these theorems in Physics and Mechanical Engineering.