MATH 2415 Calculus of Several Variables

Fall-2019

PLTL Week# 3[Sec 12.5(Planes), Sec 15.7, 15.8(Coordinates only)]

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1. Find an equation of the plane through the point $(3, 1, -4)$ and with normal vector $\mathbf{n} = -4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ in (a) vector form (b) scalar form.

2. Find an equation of the plane through the points $(3, 1, -4), (1, 2, 3)$, and $(3, 5, -7)$ in (a) vector form (b) scalar form.

3. Find an equation of the plane through the point $(3, 1, -4)$ and parallel to the plane $-4x + 3y + z = 1$. Also find the point where the line $x = 2 + 3t,\ y = -4t,\ z = 5 + t$ intersects the plane.

4. Find a vector parametrization of the plane through the points $(1, 1, 1), (2, 3, 1)$, and $(1, 0, 5)$. Also, find the scalar parametric equations of the plane.

5. Find level set equation of the plane through the point $(1, 2, -3)$ and containing the vectors $\langle 1, 2, 3\rangle$ and $\langle 3, 5, -7\rangle$ in (a) vector form (b) scalar form.

6. Parametrize the plane through the point $(1, 2, -3)$ and containing the vectors $\langle 1, 2, 3\rangle$ and $\langle 3, 5, -7\rangle$ in (a) vector form (b) scalar form.

7. Given an equation of a plane $-4x + 3y + z = 1$ in scalar form:

   (a) Write the plane as $z = f(x, y)$

   (b) Parametrize the plane. (Vector and scalar both).

   (c) Assume that you are given parametric equations of a plane as in part(b), find a scalar equation.

8. Parametrize the following planes:

   (a) $3x + 4y = 12$

   (b) $z = 2$

   (c) $y + z = 7$

9. Write a vector equation and a scalar equation of the plane with scalar parametrization
$$\begin{cases} x = 1 + 2s - 3t \\ y = s + t \qquad s, t \in \mathbb{R} \\ z = 1 + 3s - t \end{cases}$$

10. Plot the points given in cylindrical coordinates. Then convert in to the rectangular coordinates.
    (a) $\left(-2, \frac{\pi}{3}, 1\right)$ (b) $\left(\sqrt{2}, \frac{3\pi}{4}, 2\right)$

11. Convert the rectangular coordinates into cylindrical coordinates.
    (a) $(3, 3, 3)$ (b) $(-2,\ 2\sqrt{2},\ 2)$

12. Describe the surface represented by the equation in cylindrical coordinates:
    (a) $r = 3$ (b) $\theta = \frac{\pi}{4}$
    (c) $r^2 + z^2 = 9$ (d) $r = 2\cos\theta$

13. Write the equations in cylindrical coordinates

   (a) $x^2 + y^2 + z^2 - y = 4$
   (b) $2x - y + z = 1$

14. Sketch the solid E:

   (a) $E = \{(r, \theta, z) : 0 \le z \le r^2,\ 0 \le r \le 2,\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\}$
   (b) $E = \{(r, \theta, z) : 0 \le z \le r,\ 0 \le r \le 2,\ 0 \le \theta \le 2\pi\}$

15. Write the following solids in cylindrical coordinates:

   (a) $E = \{(x, y, z) : \sqrt{x^2 + y^2} \le z \le 2,\ -\sqrt{4 - y^2} \le x \le \sqrt{4 - y^2},\ -2 \le y \le 2\}$
   (b) $E = \{(x, y, z) : 0 \le z \le 4 - x^2 - y^2,\ 0 \le y \le \sqrt{4 - x^2},\ -2 \le x \le 2\}$

16. Plot the point with spherical coordinates $\left(4, -\frac{\pi}{4}, \frac{\pi}{3}\right)$. Also find the corresponding rectangular coordinates.

17. Convert the rectangular coordinates $\left(\sqrt{3}, -1, 2\sqrt{3}\right)$ in to spherical coordinates.

18. Express the following solid regions using spherical coordinates.

   (a) Unit ball E
   (b) The solid between the spheres of radius 1 and 2 centered at origin.
   (c) The solid hemisphere $x^2 + y^2 + z^2 \le 9,\ y \ge 0$
   (d) The portion of the unit ball $x^2 + y^2 + z^2 \le 1$ that lies on the first octant.
   (e) $E = \{(x, y, z) : \sqrt{x^2 + y^2} \le z \le \sqrt{2 - x^2 - y^2};\ 0 \le y \le \sqrt{1 - x^2};\ 0 \le x \le 1\}$
   (f) $E = \{(x, y, z) : -\sqrt{25 - x^2 - y^2} \le z \le \sqrt{25 - x^2 - y^2};\ -\sqrt{25 - y^2} \le x \le \sqrt{25 - y^2};\ -5 \le y \le 5\}$

19. Sketch the solid described by the given inequalities in spherical coordinates:

   (a) $\rho \le 1,\ 0 \le \phi \le \frac{\pi}{6},\ 0 \le \theta \le \pi$
   (b) $1 \le \rho \le 3,\ \frac{\pi}{3} \le \phi \le \frac{2\pi}{3}$