MATH 2415 Calculus of Several Variables

Fall-2019

PLTL-Week # 7(Sec 14.5)

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1. Let $w = f(x, y, z),\ x = x(t), y = y(t), z = z(t)$, use a tree diagram to write a formula for $\frac{dw}{dt}$.
2. Let $w = f(x, y, z),\ x = x(s, t), y = y(s, t), z = z(s, t)$, use tree diagram to write formula for $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$.
3. Let $z = f(x, y),\ x = x(r, s, t), y = y(r, s, t)$, use tree diagram to write formula for $\frac{\partial z}{\partial r}, \frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.
4. Find $\frac{dz}{dt}$
   (a) $z = x^3 + 3x^2y - 3xy^2 - y^3;\ \ x = t^2 - 2t,\ y = \frac{t^3}{3}$.
   (b) $z = \frac{x-y}{x+2y};\ \ x = e^{\pi t},\ y = e^{-\pi t}$.
   (c) $z = \sqrt{1 + xy};\ \ x = \tan t,\ y = \tan^{-1} t$.
5. Let $w = f(x, y, z) = ze^{\frac{x}{y}};\ \ x = x(t) = 1 + 2t,\ y = y(t) = t^2,\ z = z(t) = 1 - t$.
   (a) Convert w in to a function of one variable $w = g(t) = f(x(t), y(t), z(t)$, and use the chain rule for a function of one variable to find $\frac{dw}{dt} = g'(t)$.
   (b) Find $\frac{dw}{dt}$ using the Chain Rule for function on the curve $\mathbf{r}(t)$.
6. Repeat Question# 5 for $w = \ln\sqrt{x^2 + y^2 + z^2};\ \ x = \sin t,\ y = \cos t,\ z = \tan t$
7. Use the chain rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$
   (a) $z = (2x - 3y)^5;\ \ x = s^2t,\ y = st^2$
   (b) $z = \ln\left(x^2 + y^2\right);\ \ x = s\ln t,\ y = te^s$
   (c) $z = \sqrt{y}e^{xy};\ \ x = s^2 - t^2,\ y = 1 + st$
   (d) $z = e^r \sin\theta;\ \ r = st,\ \theta = \sqrt{s^2 + t^2}$
8. Let $p(t) = f(x, y)$, where f is differentiable, $x = g(t),\ y = h(t),\ g(2) = 4,\ g'(2) = -3,\ h(2) = 5,$ $h'(2) = 6,\ f_x(4, 5) = 2,\ f_y(4, 5) = 8$. Find $p'(2)$.
9. Let $R(s, t) = G(u(s, t), v(s, t))$, where G, u, v are differentiable, $u(1, 2) = 5,\ u_s(1, 2) = 4,$ $u_t(1, 2) = -3,\ v(1, 2) = 7,\ v_s(1, 2) = 2,\ v_t(1, 2) = 6,\ G_u(5, 7) = 9,\ G_v(5, 7) = -2$. Find $R_s(1, 2)$ and $R_t(1, 2)$
10. Let $w = xy + yz + zx;\ \ x = r\cos\theta,\ y = r\sin\theta,\ z = r\theta$; find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$. Also find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial \theta}$ when $r = 2, \theta = \frac{\pi}{2}$.