MATH 2415 Calculus of Several Variables

Fall-2019

PLTLWeek#8 (Sec 14.6, 14.7A)

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1. Given function $f(x,y,z) = x^2yz - xyz^3$, vector $\mathbf{u} = \frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}$, and point $P(2,-1,1)$
   (a) Find the gradient $\nabla f(x,y,z)$
   (b) Evaluate the gradient at point P, i.e. find $\nabla f(2,-1,1)$
   (c) Find the rate of change of f at point P in the direction of vector $\mathbf{u}$.
   (d) Find the direction (unit vector) in which $f(x,y,z)$ has maximum rate of change. Also find the maximum rate of change.
   (e) Find the direction (unit vector) in which $f(x,y,z)$ has minimum rate of change. Also find the minimum rate of change.
2. Repeat previous question for $f(x,y,z) = y^2e^{xyz}$, $P(0,1,-1)$, $\mathbf{u} = \langle \frac{3}{13}, \frac{4}{13}, \frac{12}{13} \rangle$
3. Find the directional derivative of the following functions in the direction of given vector
   (a) $f(x,y,z) = xy^2 \tan^{-1} z$, $P(2,1,1)$, $\mathbf{v} = \langle 1,1,1 \rangle$
   (b) $f(x,y,z) = \ln(3x+6y+9z)$, $P(1,1,1)$, $\mathbf{v} = 4\mathbf{i} + 12\mathbf{j} + 6\mathbf{k}$
4. Find the directional derivative of $f(x,y) = y\cos(xy)$ at point $(0,1)$ in the direction which makes an angle $\theta = \frac{\pi}{4}$ with positive $x-$axis.
5. Find the maximum rate of change of $f(x,y,z) = x\ln(yz)$ at point $\left(1,2,\frac{1}{2}\right)$
6. The temperature at point (x,y,z) is given by
$$T(x,y,z) = 200e^{-x^2-3y^2-9z^2}$$
   where T is measured in $^\circ C$ and x,y,z in meters.
   (a) Find the rate of change of the temperature at the point $P(2,-1,2)$ in the direction toward the point $(3,-3,3)$
   (b) In which direction does the temperature increase fastest at P?
   (c) Find the maximum rate of increase in the temperature at P.
7. Find the equation of the tangent plane to the surface $xy^3z^3 = 8$ at point $P(2,2,1)$. Also find the equation of the normal at P.
8. Repeat previous question: $x+y+z = e^{xyz}$, $P(0,0,1)$
9. Q.N#3 on the textbook exercise 14.7
10. For each function below, find all critical points. For each critical point, determine whether it corresponds to a local maximum or a local minimum or a saddle point. Find all local extrema.
    (a) $f(x,y) = x^4 + y^4 - 16xy$
    (b) $f(x,y) = x^4 + 2y^2 - 4xy$
    (c) $f(x,y) = 2xye^{-x^2-y^2}$
    (d) $f(x,y) = x^4y^2$
    (e) $f(x,y) = \sin\left(x^2y^2\right)$
    (f) $f(x,y) = x + \frac{25}{x} - y - \frac{36}{y} + 19$