Practice Problems – Final Exam

Find a parametrization of the following:

$\boxed{1}$ $x^2+y^2=9$

$\boxed{2}$ $\dfrac{x^2}{4}+\dfrac{y^2}{9}=1$

$\boxed{3}$ line segment from $(1,2)$ to $(3,5)$

$\boxed{4}$ $x^2+y^2+z^2=16$

$\boxed{5}$ $\dfrac{x^2}{4}+\dfrac{y^2}{9}+\dfrac{z^2}{25}=1$

$\boxed{6}$ $(x-5)^2+(y+2)^2=16$

$\boxed{12.5}$ — Chain Rule

$\boxed{1}$ Let C be a curve in xy plane with parametrization $\vec{r}(t)$. Given that $f(x,y) = e^{x} - y^{2}$

$\vec{r}(2) = \langle 0, 1 \rangle$ and $\vec{r}\,'(2) = \langle -2, 3 \rangle$.

Let $z = f(\vec{r}(t))$. Find $\frac{dz}{dt}$ at $t=2$

$\boxed{14.7}$ Absolute Extrema

Find the absolute maximum and minimum values of f on the set D.

$f(x,y) = x + y - xy$; D is the closed triangular region bounded by $(0,0)$, $(0,2)$ and $(4,0)$

$\boxed{14.7}$ Local Extrema

$\boxed{A}$ Find the local maximum, minimum and saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.

$\boxed{B}$ $f(x,y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 60.$

$\boxed{15.1}$

Evaluate the integral $\displaystyle\iint_R y\, e^{-xy}\, dA$; $R = [0,2] \times [0,3]$

u-substitution yields:

$$\int e^{ax}\, dx = \frac{e^{ax}}{a} + C \quad \text{if } a \neq 0$$

$$\int_e^f e^{ax+by+c}\, dx = \left. \frac{e^{ax+by+c}}{a} \right|_{x=e}^{f} \quad \text{if } a \neq 0$$

$$\int_e^f e^{ax+by+c}\, dy = \left. \frac{e^{ax+by+c}}{b} \right|_{y=e}^{f} \quad \text{if } b \neq 0$$

$\boxed{15.2}$

Evaluate the double integral $I = \int_0^1 \int_{4x}^4 e^{y^2} \, dy \, dx$

$\boxed{15.3}$

EXAMPLE 1 Evaluate $\iint_R (3x + 4y^2)\, dA$, where R is the region in the upper half-plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

$\boxed{15.6}$

EXAMPLE 2 Evaluate $\iiint_E z \, dV$, where E is the solid tetrahedron bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$.

Note :

if $a \neq 0$, then $\int_0^1 (ax+by+c)^3 \, dx = \left. \frac{(ax+by+c)^4}{4(a)} \right|_{x=0}^{x=1}$ ← u-sub

if $b \neq 0$, then $\int_0^1 (ax+by+c)^3 \, dy = \left. \frac{(ax+by+c)^4}{4(b)} \right|_{y=0}^{y=1}$

$\boxed{15.7}$

Find the volume of the solid

20. Below the cone $z = \sqrt{x^2 + y^2}$ and above the ring $1 \leqslant x^2 + y^2 \leqslant 4$

$\boxed{15.8}$

Use spherical coordinates to find the volume of the solid above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=1$

23. $\displaystyle\iint_R \frac{x-2y}{3x-y}\,dA$, where R is the parallelogram enclosed by the lines $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$, and $3x - y = 8$

16.1 - 16.2

EXAMPLE 2 Evaluate $\int_C 2x \, ds$, where C consists of the arc C_1 of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$ followed by the vertical line segment C_2 from $(1, 1)$ to $(1, 2)$.

16.1–16.2

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EXAMPLE 4 Evaluate $\int_C y^2\,dx + x\,dy$, where (a) $C = C_1$ is the line segment from $(-5, -3)$ to $(0, 2)$ and (b) $C = C_2$ is the arc of the parabola $x = 4 - y^2$ from $(-5, -3)$ to $(0, 2)$. (See Figure 7.)

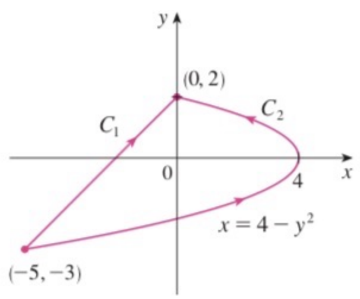

FIGURE 7

$\boxed{16.3 - 16.4}$

$\boxed{*}$ Compute $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \langle y, -x \rangle$ and

C is the unit circle oriented counter-clockwise

Use the answer to determine whether $\vec{F}$ is conservative or not.

$\boxed{*}$ Let $\vec{F}(x,y) = (3 + 2xy)\hat{i} + (x^2 - 3y^2)\hat{j}$. Is the vector field $\vec{F}$ conservative?

$\boxed{16.3}$

EXAMPLE 4

(a) If $\mathbf{F}(x, y) = (3 + 2xy)\,\mathbf{i} + (x^2 - 3y^2)\,\mathbf{j}$, find a function f such that $\mathbf{F} = \nabla f$.

(b) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve given by

$$\mathbf{r}(t) = e^t \sin t\,\mathbf{i} + e^t \cos t\,\mathbf{j} \qquad 0 \leqslant t \leqslant \pi$$

$\boxed{16.4}$

Use Green's theorem to evaluate the line integral $\int_C (1-y^3)\,dx + (x^3+e^{y^2})\,dy$, where C is the boundary of the annulus, $4 \le x^2+y^2 \le 9$. You should orient C so that the inner circle is traversed counterclockwise and the outer circle is traversed clockwise.

Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}$ is the vector field

$\vec{F}(x,y) = y\,\hat{i} - x\,\hat{j}$ and where C is the circle

$(x-2)^2 + (y+5)^2 = 9$