

# Math 2415

## Paper Homework #0

This homework set is a review of the parts of pre-calculus and calculus I that we need for MATH 2415. You may use any resources you wish. **Show all work!** Write your solutions on another sheet of paper. Make sure you understand the **theory** behind these problems, not just how to “get the answer”.

- Let  $f(x) = x^3$ .
  - Compute  $f'(2)$
  - What is the limit definition of  $f'(2)$ ?
  - What does  $f'(2)$  mean geometrically?
- Find
  - $\int_1^3 \frac{1}{x} dx$
  - $\int_1^2 (x^4 - 2)^3 x^3 dx$  [Integration by  $u$ -substitution]
  - $\int_0^3 x^2 e^{-5x} dx$  [Integration by parts]
  - $\int_0^\infty x e^{-x^2} dx$
- On what intervals is  $f(x) = 2x^3 + 3x^2 - 12x$  decreasing? Also, find the absolute minimum of  $f$  on the interval  $0 \leq x \leq 3$ .
- State two versions of the Fundamental Theorem of Calculus.
- Use two triangles and a circle to work out:  $\sin \pi/6$ ,  $\cos \pi$ ,  $\tan \pi/3$ ,  $\cot \pi/4$ . Hint: See this [Khan Academy video](#)
- Compute the equation of the tangent line to  $y = \sin(x^2)$  at  $x = \pi/6$ .
- Graph (a)  $3x + 4y = 1$ , (b)  $y = x^2 - 4x + 1$ , (c)  $x^2 - 4y^2 = 9$ , (d)  $16y^2 + 9x^2 = 1$ . *Do not use a table of values. Instead use graphing techniques you learned in your pre-calculus course. Need to brush up? See these Khan Academy videos: [Quadratics](#), [Ellipses](#), and [Hyperbolas #1](#) and [Hyperbolas #2](#).*
- The **Chain Rule** states that if  $h(x) = f(g(x))$  then  $h'(x) = f'(g(x))g'(x)$ . This means we evaluate  $f'$  at the point  $g(x)$ ,  $g'$  at the point  $x$  and then take the product of the resulting numbers. Use this way of thinking about the chain rule to solve the following problems:
  - Find the derivative of  $h(x) = \sin^4(x)$
  - Find the derivative of  $h(x) = \cos(x^2)$
  - Let  $h(x) = \ln(g(x))$  where  $g(0) = 3$  and  $g'(0) = 4$ . What is  $h'(0)$ ?
  - Suppose that  $h(x) = f(g(x))$  and that  $g(0) = 2$ ,  $f(0) = 4$ ,  $g'(0) = 2$ ,  $f'(0) = 3$ ,  $g(2) = 6$ ,  $f(2) = -3$ ,  $g'(2) = -6$ ,  $f'(2) = 7$ . Find  $h'(0)$ .
- Find the area bounded by the curves  $y = x/3$  and  $x = y^2$ .

10. In this problem, we take a curve in the  $yz$ -plane and rotate it about the  $z$ -axis to obtain a surface. Sketch the resulting surfaces in the case that the curve is (a) The circle centered at the origin radius 1, (b) The parabola  $z = y^2$ , (c) The line segment joining  $(2, 1)$  to  $(3, 2)$ .
11. Use the techniques described in the [Surface Sketching Primer](#) to sketch
- (a) A square in the horizontal plane,  $z = 3$
  - (b) A rectangle in the vertical plane,  $x = 2$
  - (c) A parallelepiped
  - (d) A cereal bowl with a circular rim
  - (e) A cylinder aligned with the  $z$ -axis
  - (f) An icecream cone with a spherically shaped scoop of icecream in the top