

**Math 2415**  
**Paper Homework #12**

**1. 15.7, Triple Integrals in Cylindrical Coordinates**

- (a) Let  $E$  be the solid region in the first octant (*i.e.*, where  $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ ) that is inside the cylinder  $x^2 + y^2 = 1$  and below the plane  $x + z = 1$ . Calculate  $\iiint_E y \, dV$ . **Hint:** This problem was in PS #12.
- (b) Find  $\iiint_E \sqrt{x^2 + y^2} \, dV$  where  $E$  is the solid region above the  $xy$ -plane, below the paraboloid  $z = 8 - x^2 - y^2$  and *outside* the cylinder  $x^2 + y^2 = 1$ . Sketch the solid  $E$ .

**2. 15.8, Triple Integrals in Spherical Coordinates**

- (a) Find  $\iiint z \, dV$  where  $E$  is the solid region that is inside the sphere  $x^2 + y^2 + z^2 = 4$  and above the cone  $z = \sqrt{x^2 + y^2}$ . **Hint:** This problem was in PS #12.
- (b) Find the volume of the region inside the ball  $x^2 + y^2 + z^2 \leq R^2$  that lies between the planes  $y = 0$  and  $y = \sqrt{3}x$  in the first octant.

**3. 15.9, Change of Variables Theorem:**

- (a) Evaluate  $\iint_R (x - y)^2 e^{x+y} \, dx \, dy$  where  $R$  is the parallelogram bounded by  $x + y = 1$ ,  $x + y = 3$ ,  $x - y = -2$  and  $x - y = 1$ . **Hint:** This problem was in PS #12.
- (b) Use the change of variables  $u = y/x^2$ ,  $v = x/y^2$  to find the area of the region in the first quadrant that is bounded by the curves  $y = x^2$ ,  $y = 3x^2$ ,  $x = y^2$  and  $x = 4y^2$ .